GROUPS

DIMENSIONS OF MATHEMATICS

Groups

Mark Cartwright

M
MACMILLAN

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*To Leslie,
for understanding*

CONTENTS

Preface	ix
1 NUMBERS, VECTORS AND MATRICES	1
1.1 The natural numbers	1
1.2 The integers	5
1.3 The rationals	7
1.4 The reals	9
1.5 Vectors	11
1.6 Matrices	14
1.7 The integers modulo n	19
1.8 The integers modulo p	22
2 PERMUTATIONS	25
2.1 Shuffling shells	25
2.2 Permutations	26
2.3 Composition of permutations	28
2.4 Systems of permutations	32
3 SYMMETRY	37
3.1 Symmetry of plane figures	37
3.2 Types of symmetry	39
3.3 Symmetries and matrices	42
3.4 Arithmetic on symmetries	47
3.5 Systems of symmetries	50
3.6 Solids	52
3.7 The Platonic solids	55
3.8 Rotational symmetries	56
3.9 Tilings	60

4	GROUPS	64
4.1	Axioms	64
4.2	Examples of groups (and non-groups)	67
4.3	Cayley tables	70
4.4	Isomorphism	75
5	INSIDE GROUPS	79
5.1	Powers	79
5.2	Generators	82
5.3	Cyclic groups	85
5.4	Subgroups	87
5.5	Finding subgroups using Cayley tables	90
6	THE CLASSIFICATION PROGRAMME	95
6.1	Why classify groups?	95
6.2	Using the Latin square property	98
6.3	Using groups we know about	101
6.4	Isomorphic tables	103
6.5	Failed groups	106
7	COUNTING COLOURED SHAPES	111
7.1	Black and white tiles	111
7.2	The group approach	113
7.3	Justifying the method (i)	115
7.4	Justifying the method (ii)	119
7.5	A three-dimensional example	121
	Glossary	124
	Index	129

PREFACE

It is not easy to decide what to put in a volume like this; or, having decided what to include, *how* to put it in. What follows is, then, a very personal approach to introducing groups and group theory.

To justify it, I would say that the real stumbling block that students find in approaching the subject is its abstract nature. I have therefore included a fairly extensive discussion of examples before identifying them as groups, the hope being that this stock of examples might provide a 'map of the land'. I have also presented a lot of the 'formal' theory from the point of view of (comparative concrete) Cayley tables.

There will be people who will wish that I had introduced (say) the cycle notation for permutations, or proved Lagrange's theorem. To these I say: there is plenty of time for those things in a university course. This book is all about what groups *are*, and the frame of mind in which to approach them.

The final chapter is, I admit, a bit courageous: a development of (only a slight bowdlerisation of) the orbit-stabiliser theorem. This is here because of its great practical relevance, very rare in elementary group theory. It is possible to do this without introducing cosets or a formal idea of group actions.

Finally, I must commit to print my immense debt of gratitude to my wife Leslie, for reading the manuscript and (most encouragingly) liking it. Thanks too go to the series editors and to the staff at Macmillan for their support and guiding hands.