WAVES

DIMENSIONS OF MATHEMATICS

Waves

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PREFACE

This book provides an introduction to the phenomenon of wave motion. The material would usually be covered in an elementary course in applied mathematics at undergraduate level. However, it has been written so that it should be accessible to upper sixth form pupils who are studying mathematics and who wish to extend their interest further.

The only mathematical assumption that I have made is that the reader is familiar with the techniques of algebra and calculus usually found in 'A' level pure mathematics. Consequently, some parts of the text are developed in a manner which is mathematically less elegant than otherwise would be the case, e.g. I make no reference at all to partial differentiation in relation to the wave equation, nor to eigenvalues in relation to normal modes.

As far as the applied mathematics is concerned, I have assumed that the reader is familiar with Newton's second law. A knowledge of simple harmonic motion would be very useful, but has not been assumed.

Throughout the text, the reader is encouraged to develop some of the results via *PFTAs*: '*pauses for thought and action*'. These exercises form an important part of the learning process and as such involve the reader in an active rather than a passive role.

However, it is quite possible to read the text without attempting all of the exercises. Where appropriate, I have included simple experiments to illustrate more clearly the ideas being discussed. These can be performed at home using easily available equipment. Also, I have indicated where the reader with a personal computer could produce simple programs to illustrate some of the more complicated results. There are many programming languages now available on PCs and so I have avoided being specific by giving the program segments in pseudo-code. In the pseudo-code segments, the command `set_polyline (n, x, y)` is the command to draw the set of straight lines joining the n points whose Cartesian coordinates are given in the arrays x and y . The interested reader should have little difficulty producing programs in any suitable language.

PREFACE

Finally, I hope that the book provides a stimulation to find out more about waves. The bibliography provides a short list of textbooks and articles which could form a source for further study of wave motion.

Alan Davies