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MACMILLAN WORK OUT SERIES

Fluid Mechanics

G. Boxer

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MACMILLAN

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Preface

This is a revision tutorial book like its companion volumes in the Work Out Series. It is intended to be used in the period between the end of lectures in the early stages of a degree course and the start of sessional examinations. In certain cases a fairly full summary of fundamental lecture material is incorporated as students will want to compare the nomenclature in use here with that in their own notes or textbook.

A knowledge of elementary differential and integral calculus on the part of the student is taken for granted and the solutions are effected in the *Système International* since this is now in common use throughout the United Kingdom and on the European continent.

The fundamentals of fluid mechanics and engineering thermodynamics bear a close relationship to each other and the principles of conservation of mass, energy and momentum apply in each subject in very similar fashion. Indeed, attempts have been made elsewhere to combine the two subjects at first-year level to demonstrate this essential unity, but without real success. Where the two subjects are inextricably linked (e.g. in one-dimensional steady-flow gas dynamics – Chapter 10 in this book) the commonality is clearly depicted. Indeed, it is not until Chapter 10 that temperature becomes a significant variable and thermodynamics becomes an essential part of the analysis.

I have attempted wherever possible to use a nomenclature common to both thermodynamics and fluid mechanics, since they are companion volumes in this series and the two go hand in hand in any degree course in mechanical engineering.

The subject matter chosen should cover all that is likely to be included in the early stages of a degree course and the book is completed by the full solution of two sessional examination papers (as in *Work Out Engineering Thermodynamics*), so that students are forced to make their own decision concerning the nature of the question and the particular fundamentals involved without the prior benefit of chapter headings.

I have tried in the Introduction to stress the vital importance of a logical approach to all problems and the correspondingly vital need to maintain a correct dimensional balance in all reasoning. This cannot be too strongly emphasised. A great many difficulties can be resolved if the student forms the right habits from the word go.

I must pay tribute to two former colleagues of mine at the University of Aston for permission to include a considerable proportion of the material used in this book, namely Mr W. H. Kinsman and Mr J. C. Scouller.

The tutorial work presented herein has been used over a long period of time at Aston and is fully tested in the classroom. However, if any errors are discovered I will be very grateful if they are notified to me.

1988

G. B.

Nomenclature

<i>A</i>	area
<i>A</i>	location
<i>a</i>	dimension
<i>a</i>	atmospheric, acoustic or air (subscript)
<i>B</i>	dimension
<i>B</i>	location
<i>b</i>	dimension
<i>C</i>	centre line
<i>C</i>	Chezy constant or coefficient ($C_D/\sqrt{1 - (A_0/A_1)^2}$, where C_D = Discharge coefficient)
<i>c</i>	specific heat capacity
<i>c_p</i>	specific heat capacity at constant pressure
<i>D</i>	dimension
<i>d</i>	dimension
<i>d</i>	differential operator or location
<i>D</i>	location or discharge
<i>E</i>	internal energy
<i>e</i>	specific internal energy ($= E/m$)
<i>e</i>	location or equivalent (subscript)
<i>F</i>	force
<i>f</i>	friction factor
<i>f</i>	friction or (as a subscript) liquid state
<i>G</i>	centre of gravity
<i>g</i>	gravitational acceleration
<i>g</i>	saturated vapour state (as a subscript)
<i>H</i>	horizontal (subscript)
<i>H</i>	enthalpy or head
<i>h</i>	specific enthalpy ($= H/m$) or head or depth in a fluid
<i>I</i>	second moment of area
<i>I_O</i>	second moment of area about the centroid
<i>i</i>	inlet (to a pump)
<i>i</i>	slope of a channel
<i>K</i>	bulk modulus of elasticity
<i>K</i>	Kelvin (unit of temperature on the absolute scale)
<i>L</i>	length
<i>l</i>	length
<i>M</i>	fundamental dimension for mass
<i>m</i>	manometric or mean (subscript)
<i>m</i>	mass
<i>m_v</i>	relative molar mass
<i>ṁ</i>	mass flow rate ($= dm/dt$)

N	rotational speed
n	polytropic index
O	orifice
o	oil or conditions at solid boundary (subscript)
P	perimeter
p	pressure
P	pump, piston or plunger
Q	heat transfer
$\dot{Q}$	heat transfer rate ($= dQ/dt$)
q	specific heat transfer ($= Q/m$)
R	Rankine unit of absolute temperature
R	radius or specific gas constant
R_0	universal gas constant
r	radius
S	entropy or specific gravity
s	system
s	specific entropy ($= S/m$) or distance
t	time or thickness (of an oil film)
t	throat
T	temperature
T	turbine (subscript)
u	velocity
V	volume
$\dot{V}$	volume flow rate ($= dV/dt$)
v	specific volume ($= V/m$)
V	vertical (subscript)
W	work transfer or weight
$\dot{W}$	rate of work transfer (i.e. power) ($= dW/dt$)
w	water (subscript)
w	specific work transfer ($= W/m$)
x	principal coordinate, distance or quality (dryness) of vapour
y	principal coordinate or dimension
Y	dimension
z	datum height (as in potential energy per unit mass gz)
α	angle
γ	isentropic index
δ	increment in (e.g. δx)
ϵ	excrescence height (average) in rough pipes
η	efficiency
θ	angle
μ	coefficient of absolute viscosity
ν	kinematic viscosity ($= \mu/\rho$)
π	as in area of circle (i.e. πr^2)
ρ	density
τ	shear stress
ω	angular velocity
Δ	increment in (e.g. Δx)
σ	surface tension
ϕ	function of
(Re)	Reynolds number ($= \rho u L / \mu$)
(Ma)	Mach number ($= u / u_a$)
(Fr)	Froude number ($= u^2 / gL$)

head coefficient = $(g \Delta z)/u^2$

drag coefficient = $F_D/(0.5\rho u^2 L^2)$

lift coefficient = $F_L/(0.5\rho u^2 L^2)$

Other symbols

∂ partial differential operator

$\propto$ proportional to

$>$ greater than

$<$ less than

$\cdot$ maximum value (e.g. $\hat{W}$)

∞ infinity

Σ sum of

$\equiv$ equivalent to

Introduction

General Approach to Problem Solving

Fluid mechanics deals with the study of fluids (liquids and gases), both at rest (statics) and when flowing (fluid dynamics). The need to study fluid mechanics is paramount for the engineer and the applications are numerous and very varied. The flow of oil in a pipeline, of blood through a human body, of air round a propeller are but three examples of fluid dynamics. The forces on a masonry dam, the study of manometry and the use of hydraulic pressure to bend pipes are three obvious examples of fluid statics.

Very often the engineering problem involves the principles embodied in both fluid mechanics and thermodynamics and the trained engineer has learned to handle both simultaneously. However, having read and heard the arguments for a combined treatment of the two subjects at first-year level, I feel (as already stated in the Preface) that there is probably more to be gained by separate treatment at first, with appropriate cross-reference, leaving the dovetailing of the subjects to the more mature student. One obvious cross-reference is the link between the steady-flow energy equation and Bernoulli's equation which is demonstrated later in this book.

Like thermodynamics, the subject of fluid dynamics is often, misguidedly, subdivided in the student's mind into a series of discrete packets of theory, e.g. flow through venturimeters, manometry, and water meters, each containing its own individual formulae. In thermodynamics a comparably erroneous approach would similarly segregate piston engines, gas turbines, steam engines, and so on.

The student who wishes to master both subjects must learn to recognise that the same fundamental concepts are used repeatedly in both subjects, and that when these are absorbed the subsequent development of each is manifestly easier.

I have tried to emphasise this by postulating certain basic questions which should be asked every time a problem is attempted. I do not maintain that the questions are relevant to every problem encountered in engineering, since the range is so wide (there are obvious exceptions such as in manometry where they are of a limited assistance), but very often they give a clear guideline to a possible solution.

The first two questions are interrelated, but are given separately in the hope that students will be helped to a quicker understanding of the nature of the problem than they would be if the two questions were combined into a rather longer and more complex one.

(a) What Kind of *Process*?

In the subject of fluid mechanics an important distinction is between steady flow of a fluid (flow in which it may be assumed that properties across the cross-section of the fluid are constant with time, and mass flow rate is fixed), unsteady flow (mass flow rate a variable), and fluid statics (as in manometry).

(b) What Kind of *Fluid*?

In fluid mechanics the important distinction is made between incompressible (i.e. constant-density) fluids and compressible (variable-density) fluids. There will be occasions when fluids which are normally regarded as compressible, e.g. gases, are taken as incompressible since density changes are small (e.g. air flow round a lorry).

Furthermore, in the case of water-hammer, the liquid must be considered as compressible (whereas water is normally regarded as incompressible, especially in the problems in this elementary book) since the flow is characterised by the propagation of waves of finite amplitude with finite velocity.

(c) Have You Drawn a *Sketch* of Events?

Engineers work with pictures because they afford a valuable insight into the problem under examination. In thermodynamics the obvious picture is a field of state (e.g. pressure versus specific volume) and this may also be helpful in fluid mechanics, but any relevant diagram will be helpful.

(d) Do You Need *Mass Continuity*?

This is $\dot{V} = \dot{m}v = uA,$

where the symbols have the meaning listed in the nomenclature (pp. viii–x).

(e) Do You Need the *Energy* Equation?

This question is bound up with (a) and (b) above. For example, Bernoulli's equation may well be an essential part of the solution since this is a statement of the conservation of mechanical energy. However, for compressible fluid flow, where temperature proves to be a significant variable, the full steady-flow energy equation will be required since this shows the conservation of the sum of both mechanical and thermal energies.

(f) Do You Need the *Momentum* Equation?

As in *Work Out Engineering Thermodynamics*, I have included a reasonably full treatment of this, since students experience so much trouble with it on first meeting it. Momentum ideas are involved whenever forces are to be calculated, whether from the application of fluid pressure, shear forces and body forces or from acceleration or deceleration of fluids or both.

(g) Have You Used the Correct *Language*?

For example, w for specific work transfer (in kJ/kg), W for work transfer (in kJ) and $\dot{W} = dW/dt$ = rate of doing work (or power) (kW), since the dimensions of a differential coefficient are determined by drawing a line, viz. $\frac{d}{dt} \left| \frac{W}{t} \right|$, and considering the dimensions to the right of the vertical line i.e. in this case W/t in kJ/s or kW.

(h) Have You Ensured Correct *Dimensional Reasoning*?

This is, of course, allied to (g) above since correct language goes hand in hand with correct dimensions.

Example 1

From hydrostatics we have the hydrostatic equation:

$$p = \rho gh$$

and for $\rho = 10^3 \frac{\text{kg}}{\text{m}^3}; \quad g = 9.81 \frac{\text{m}}{\text{s}^2}; \quad h = 2 \text{ m};$

i.e. $p = 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 2 \text{ m} = 19\,620 \frac{\text{kg}}{\text{m s}^2},$

which is not a readily recognisable unit of pressure.

However, by using unity brackets we can ensure that the answer comes out in obvious units of pressure. Thus in Newton's law we have

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/s}^2$$

or
$$\left[\frac{\text{N s}^2}{\text{kg m}} \right] = \left[\text{UNITY} \right] = \left[\frac{\text{kg m}}{\text{N s}^2} \right],$$

and anything multiplied by unity (or its reciprocal) is unchanged.

Thus in the above calculation we may write

$$p = 19\,620 \frac{\text{kg}}{\text{m s}^2} \left[\frac{\text{N s}^2}{\text{kg m}} \right] = 19\,620 \frac{\text{N}}{\text{m}^2} = 19\,620 \text{ pascals} \\ = 0.1962 \text{ bar}.$$

This gives the correct answer in the right units even though there is no numerical change in this instance. In the next example the necessary introduction of two unity brackets does bring about a substantial change in the numerical answer in order that the right units may appear.

Example 2

From the steady flow of water through a horizontal nozzle from negligible velocity and a pressure of 50 bar to an exit pressure of 40 bar where $\rho = 10^3 \text{ kg/m}^3$, we have from Bernoulli's equation for incompressible flow:

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} = \frac{p_2}{\rho} + \frac{u_2^2}{2},$$

and since u_1 is given as very small then

$$u_2 = \sqrt{\frac{2(50 - 40) \text{ bar}}{10^3 \frac{\text{kg}}{\text{m}^3}}} = \sqrt{\frac{2}{10^2} \frac{\text{bar m}^3}{\text{kg}}},$$

which is not recognisable as a velocity.

Now, by introducing two unity brackets we rationalise the result. Thus

$$u_2 = \sqrt{\frac{2}{10^2} \frac{\text{bar m}^3}{\text{kg}} \left[\frac{10^5 \text{ N}}{\text{bar m}^2} \right] \left[\frac{\text{kg m}}{\text{N s}^2} \right]} = \sqrt{2000 \frac{\text{m}^2}{\text{s}^2}} = 44.72 \frac{\text{m}}{\text{s}}.$$

(Note that without dimensions the answer would have come out as 0.1414 with a guess at the dimensions!)

Note: Although the SI unit of pressure is the pascal (Pa) ($= 1 \text{ N/m}^2$), in this volume use is made of the bar ($= 10^5 \text{ N/m}^2$) and its multiples and submultiples because this is such common practice elsewhere, for example the millibar in weather forecasting.

Throughout this volume the fundamental ideas are given emphasis in the text to make them stand out and in the two examination papers at the end an attempt is made to apply the above questions wherever possible to show their recurring relevance.

Some General Definitions in Fluid Mechanics

Ideal Fluid

This is assumed to have zero viscosity (for a fuller definition see Chapter 1). There is no conversion of mechanical energy into thermal energy with an incompressible ideal fluid. If an ideal fluid is initially at rest, it can be shown that, subsequently, all particles must continue to have the same total mechanical energy.

Real Fluid

Viscosity is unavoidable in a real fluid; flow of a real fluid involves the conversion of mechanical energy to thermal energy.

Laminar Flow (or Streamline or Viscous Flow)

In this case, fluid particles move along in layers or laminae with one layer sliding over an adjacent layer. The flow is governed by Newton's law of viscosity (for one-dimensional flow). (See Chapter 1 for further treatment.) The degradation in mechanical energy varies approximately as (velocity)¹.

Turbulent Flow

This occurs when the fluid particles move in very irregular paths causing an exchange of momentum from one portion of a fluid to another. The degradation in mechanical energy varies approximately as (velocity)².

Laminar flow tends to occur when the fluid velocity is small or the fluid viscosity is large or both. In cases where the given flow can be either laminar or turbulent, the turbulent flow sets up greater shear stresses and causes more mechanical energy to be converted to thermal energy.

Steady Flow

This is characterised by a steady mass-flow rate and by the fact that across any section at right angles to the flow all properties are constant with respect to time. True steady flow is found only in laminar flow.

Steady, turbulent flow is said to exist when the *mean* velocity of flow at a section remains constant with time.

Unsteady Flow

This occurs when conditions at any point change with time. An example is the flow of a liquid being pumped through a fixed system at an increasing rate.

Uniform Flow

This occurs when, at every point, the velocity vector is identical in magnitude and direction at any given instant. That is, $\partial u / \partial s = 0$, in which time is held constant and s is the displacement in any direction. This equation states that there is no change in the velocity vector in any direction throughout the fluid at any instant. It states nothing about the change in velocity at a point with time.

In the flow of a real liquid in an open or closed conduit, the definition may be extended in most cases, even though the velocity vector at the boundary is always zero. When all parallel cross-sections through the conduit are identical (i.e. when the conduit is prismatic), and the average velocity at each section is the same at any given instant, the flow is said to be uniform.

An example is the flow of a liquid being pumped through a long straight pipe (uniform in the direction of flow).

Non-uniform Flow

This occurs when the velocity vector varies from place to place at any instant, i.e. $\partial u / \partial s \neq 0$. An example is the flow of a liquid through a tapered or curved pipe.

Other Examples

Steady, uniform flow The flow of a liquid through a straight horizontal pipe at a constant rate.

Unsteady, uniform flow The flow of a liquid through a straight horizontal pipe at a changing rate.

Steady, non-uniform flow The flow of a liquid through a conical pipe at a constant rate.

Unsteady, non-uniform flow The flow of a liquid through a conical pipe at a changing rate.

Path Line

This is the path of a single fluid particle taken over a length of time. Path lines may cross each other.

Streamline

This is a curve which is everywhere parallel to the direction of flow. No streamlines can ever cross each other. Streamlines and path lines become identical if the flow does not change with time, i.e. if the flow is steady. Streamline pictures allow fluid flow to be visualised.

Stream Tube

The total flow can be divided into parts, each taking place along an imaginary tube, called a stream tube. A stream tube is bounded by streamlines and hence no flow can cross the boundary of the stream tube. If the fluid is ideal (i.e. if there is no energy degraded) then the total mechanical energy per unit mass of fluid remains constant along a stream tube (i.e. the flow is isenergetic).

The product of velocity and cross-sectional area is assumed constant for all stream tubes when drawing streamline fields for an incompressible fluid.

Boundary Layer

When a real fluid flows past a solid boundary, the fluid in the immediate neighbourhood of the boundary is retarded by viscous shear forces set up within the fluid. The retarded layer of fluid is called the boundary layer.

Continuum

There are two ways of approaching problems in fluid mechanics.

In the *microscopic* approach the individual molecular motion is examined mathematically.

In the *macroscopic* approach (used in this volume) the behaviour of the fluid is described mathematically by examining the motion of small volumes of the fluid which contain a large number of molecules. This implies that since we are taking average effects of many molecules the fluid is being regarded as a continuum (i.e. as a continuous medium). This is justified if the characteristic dimension is very much greater than the mean free path of the molecules.