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MACMILLAN
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Dynamics

G. E. Drabble

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MACMILLAN

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The authorities accept no responsibility for solutions to the questions used, or for the correctness of the answers.

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Examinations

Some General Remarks

Although you have embarked on a course with the object of learning the theory and practice of engineering, you cannot afford to underestimate the importance of the annual examination. Notwithstanding various attempts by authorities to use other methods of assessing performance, such as continuous assessment, project work and the like, you are not going to qualify until you have shown that you can sit in a room for a few hours and show some reasonable competence in solving engineering problems. Mistrust those lecturers who tell you, 'Learn your subject thoroughly and the examination will take care of itself.' Such advice is likely to come either from brilliant young men and women who never had any trouble passing their own examinations or from old greybeards who can't even remember what taking an examination was like. Students given such advice might well be tempted to retort, 'Get me through this examination and I'll take care of the learning afterwards.'

Few people really like the examination system. Academics devise more or less abstract questions, often so contrived as to be removed almost completely from the real world of engineering, and throw them without warning at a class of students, incarcerated for three hours in a room for the purpose of trying to answer them. The experience bears practically no relation to the student's subsequent career. Studies have shown that there is little correlation between academic prowess and professional excellence. In spite of all this, there is a general consensus that there are certain things that would-be engineers should know, and that before they are allowed to work professionally they should demonstrate that they know these things by answering questions set by experts.

It is perhaps unfair to state that the best guarantee of passing an examination is to have done well in the examination previously. Nothing dogs a student's progress through a course more than weakness in certain areas at the outset. It is valuable to know that when a board of examiners meets for the purpose of deciding which students shall pass, and which fail, their prime consideration is not the marks scored in individual papers but the potential of the student (on the basis of those marks) for undertaking the work of the next year of the course. (For this reason, a final-year board is likely to be more lenient in its judgements than a first- or second-year board.) After passing a first-year examination a student will sometimes ask his tutor, 'What work do you advise me to do to prepare for next year?' My own answer to this question was always, 'Go through all the papers you have just taken, and make sure you can answer *every* question, not just five out of eight. Work on your weaknesses now, while you have a short respite before the next year begins.' Each year of the course builds on the work of the previous year.

However, we have to face the facts, and begin with how we find things at the moment, not how we should have them arranged if we could relive the past few

years. Whatever your previous history and performances, you will have to face an examination at the end of the year. Set out below are some suggestions. Some are no doubt obvious; some perhaps not so obvious; some even might be of no use at all to you personally. The suggestions fall into two clear categories: what to do before the examination and what to do during the examination.

Before the Examination

1. If you are taking an internal examination, find out who the examiners are for your subjects. The subject teachers themselves are usually the examiners. For an external examination, you will not know who the examiners are and will not need to know. (An external examination is one which is set by a Board of Examiners who have nothing to do with the running of the course. The Ordinary and Advanced Level examinations of the General Certificate of Education are the most familiar examples.)
2. Obtain copies of previous examination papers in your subjects for the last two or three years. This will help to give you a 'feel' for the type of question you may expect to have to answer. But find out if the structure of the course has changed recently; if it has, then earlier papers might give you a misleading impression.
3. Check with your examiners that the conditions will be the same as for previous years, and, if they have changed, find out in what particulars.
4. Study the rubric of earlier papers carefully. The rubric is the name given to the general instructions at the head of the paper. This may vary from:

'Answer five questions'

to:

'Answer ALL of Section A; not more than TWO questions from Section B, and not more than THREE questions from Section C. Marks appropriate to each question are shown alongside the question. Candidates are required to satisfy the examiners in ALL sections of the paper. Candidates are expected to spend not more than 40 minutes in answering Section A.'

You can see that if you go into an examination and are unexpectedly faced with the almost legalistic verbosity of the second example, you may feel with some justification that you are entitled to some marks just for working out exactly what you are required to do!

5. Check that you will be allowed to use your own calculator, and whether there are any restrictions on, say, programmable calculators.
6. Find out what would be the consequences of your failing to attend an examination because of illness.
7. If you suffer from a chronic complaint, such as hay fever, or migraine, let your examiners or your tutors know, well before the time of the examination. Supplement your information with a note from your doctor. Excuses presented after an examination, even when genuine, are not likely to be accepted with sympathy; the examiners would have a tendency to suspect that the student kept the knowledge 'up his sleeve' in case he didn't do too well.
8. Shortly before the examination you might find it useful to decide on a strategy. You may tell yourself that it is quite unrealistic to revise every single topic effectively and that you will have to decide which topics to go for, and which to 'ditch'. A survey of earlier papers might help here. You may find that a question on, say, velocity diagrams has turned up every year for three years, and you may therefore, with some confidence, classify this topic as a 'banker'.

Make a list of all the topics on which questions could be set. If the paper normally contains eight questions, and you have covered only eight topics on the course, you are in the happy position of knowing with a fair degree of confidence that a question will turn up on any topic you choose. If on the other hand, twelve topics have been covered on the course, a simple calculation will tell you that in order to be sure of answering three questions, you must revise at least seven topics.

But a warning is necessary here. Some topics — a good example is work, energy and power — are sufficiently well-defined to justify the setting of a question. But such a topic is so fundamental that questions set in other topics require a knowledge of the principles (questions on momentum, for example). You should not forget that ‘topics’ are convenient subdivisions for lecturers, students and examiners (and writers of textbooks) for breaking down a subject, but that engineering problems very often cut across these rather artificial dividing lines. Many of the questions in this book will show this. So if you have to ‘write off’ some of your syllabus, then exercise considerable care in doing so.

9. It is not unknown nowadays for formula sheets to be issued in examinations, but it is certainly not universal practice. If you know that you will not receive such a sheet, part of your last-minute revision should be listing all the formulae you will need to know, and memorising them. This may not be easy for you. It will be easier, of course, if you have done a fair amount of work during the year on the subject. By the time you have reached this stage, certain basic formulae such as centripetal acceleration ($a = \omega^2 r = v^2 / r$) and kinetic energy ($\frac{1}{2}mv^2$; $\frac{1}{2}I\omega^2$) should form part of your ‘alphabet’ of engineering. A working knowledge of dimensions (a topic that students tend to shun) also helps to avoid such mistakes as confusing the surface area of a sphere ($4\pi R^2$) with the volume of a sphere ($\frac{4}{3}\pi R^3$).

But sheer memorising is difficult for some people. There are techniques which are really helpful in improving memory. But engineering students usually have so much work to do that they cannot afford the time to take up anything not directly related to the course. You should recognise that learning is not a passive business; you do not learn very effectively when sprawling in a comfortable chair, just reading your notes. To memorise a set of formulae — the contents of one of the Fact Sheets of this book, for example — first copy down on a sheet of paper all the items you have to recall, in words, leaving out the actual formula: for example:

Moment of inertia of a uniform disc =

Moment of inertia of a uniform rod about one end =

Moment of inertia of a uniform rod about centre =

When you have copied everything down, close the book, and try and fill in all the appropriate formulae, referring back to the book when you can’t remember. Keep on doing this until you can write them all down correctly. (See item 3 in the section ‘During the Examination’.)

10. Very shortly before the examination (say the night before), assemble all the necessary gear (pens, instruments, calculator, etc.) taking care to ensure that nothing contains any ‘incriminating matter’, such as a set of formulae stuck on the back of a ruler, and that your calculator is fully charged. If possible, include a watch or desk clock in your equipment.

During the Examination

Rules do vary from one institution to another. It is possible, though unlikely, that some of the hints that follow might be unacceptable in some places. If there is any possibility of doubt, check with your authority beforehand.

1. Take into the examination room only those items you are permitted to take. If you have used a formula sheet for last-minute revision, be sure to leave it outside. If by mistake you do take it in, declare the fact immediately to the invigilator.
2. Everything you write in the examination room should be written in the answer book, or on the paper provided. Anything you clearly cross out will not be read or marked by the examiner. You should therefore *not* require a sheet for rough work. Do all rough work and notes in the answer book, and cross it out when done with. **DO NOT** tear it out.
3. You may have particular trouble remembering a long and complex formula which you are sure you will need; you may have it written down on a sheet of paper outside the examination room, feverishly trying to memorise it. As soon as you are told to begin the examination, but not before, you may write the formula down into your answer book (if you can still remember it) even before you read the question paper. You then have it in front of you, whether you need it or not.
4. Now read the question paper carefully, first making sure that the rubric is of the expected form, and noting if it is not. Read every question, fairly quickly, to identify the topic covered, and also to make sure you have the full and correct complement of questions. (It is not unknown for some candidates to receive a question paper with one page blank.) If you find a question you were hoping to find, make a mark by it, as one that you are eventually going to attempt, but do not start to answer it until you have read through the paper. Some examiners allow ten minutes or so of 'reading time' added on to the statutory time for answering. If this is allowed, remember that reading-time is just that, and you may not write or make notes during that time.
5. Having read all the questions, you should have found some that you can attempt. Divide your allotted time by the number of questions you are asked to answer, thus calculating the approximate time allocation per question. This is usually about 30 minutes. Picking your question, read it through again, this time very carefully, making sure exactly what you are being asked. Candidates frequently come to grief on an answer by failing to answer the question asked, substituting instead an answer to a question they thought they should have been asked.
6. Try to keep your work neat. This is not so much for the benefit of the examiner (although he or she will be grateful) as for yourself. Mistakes in algebra and arithmetic tend to occur much more often with untidy work than with neat work. Needless to say, you should not leave it until this point to start being neat; it is a habit you should try to acquire right at the beginning. Remember that every slip will cost you marks.
7. Don't, however, be *too* neat. Orderly working, well spaced out, is a great help towards correctness. But under-ruling all subsidiary answers in green ink, and double-under-ruling the final answer in red ink will add nothing to the accuracy of the solution, and will earn you no more marks. If you have developed a habit of doing this, or something similar, try to break the habit. For the same reason if you write something you know is wrong, cross it out. Cross it out carefully and neatly by all means; a fierce scribble with the point of a ball-pen, penetrating two other pages, serves no purpose except to tell the examiner that the candidate is under stress. But *do not* carefully open a bottle of white

- opaque ink and meticulously paint out the offending item. This again takes up valuable time, and earns no marks, quite apart from being somewhat expensive.
8. As a general, though not universal rule, solve your questions as much as possible in algebra, substituting arithmetical values at the end. This has three advantages. First, an algebraical error is more easily spotted than an arithmetical one. Second, you can reduce calculator work to a single operation at the end, instead of six or seven mini-calculations throughout the solution. Third, transferring long numbers from one line of working to the next can introduce error. Suppose you calculate the second moment of area of cross-section of a shaft as $0.000\,000\,076\,49\text{ m}^4$ (a not unlikely value). Even if you call this 7.649×10^{-8} , it is still a long and tedious item to carry through a calculation; much more cumbersome than $(\pi/32)D^4$. The worked examples in this book will repeatedly illustrate this point.
 9. Show all your working in your written solution. If you do all your calculations on a sheet of blotting-paper and only set down the final result your examiner might reasonably assume that you copied it from someone else.
 10. Do not answer what you have not been asked. Read the question carefully. 'State the formula for . . . ' means just that; you don't have to prove it, and if you do prove it, you won't get marks for the proof. On the other hand, 'Derive a formula for . . . ' means that you have to show all the working, i.e. prove it.
 11. If the numerical answer to a question is clearly and obviously absurd – for example, if the deflection of a steel bar is several hundred metres, or the calculated speed of a car is several kilometres per second – leave it at this stage. It is probably a small slip, involving some carelessness with high powers of 10, and it may not cost you more than a mark or two. To go back over the whole calculation to try and retrieve the error might cost far more in precious time than the possible small gain of a mark or two justifies. If you have spare time at the end by all means come back and try to locate the mistake. But during the examination try to keep to the schedule of so many minutes per question.
 12. If you find yourself in a tangle with a question that you were absolutely confident of being able to answer, and your allotted time is running out, then *leave it*, and try another question. Avoid getting angry and frustrated, if you can; this is likely to lead to further mistakes. Keep cool, and again, if you have time at the end, then come back and have another try, preferably starting from the beginning again.
 13. You may reach a desperate stage. You may have limped through two questions, pulling in possibly 30 marks, and you know you need a minimum of 40 to pass, and you can't see another question that you can attempt with any confidence. You have come to the unenviable point where you have to pick up a few marks where you can. Many questions are set in two parts. The first part might consist of deriving a special formula and the second part, a substitution of numerical values in the formula. Well, you may feel incapable of deriving the formula, but you can substitute the numbers in it. If you do so, you won't get many marks for it; the examiner probably allots 16 marks to the first part, and only 4 to the second. But in your position, you can use those four marks, so you grab them where you can. They might mean the difference between a pass and a fail.
 14. An excellent and almost universal rule is to adhere to the instructions in the rubric. If you are required to answer five questions, do not attempt six or seven. There was a time when an examiner would mark the statutory number of answers, in the order in which they were written in the answer-book, and then *not mark any further work*. Thus, an examinee might gain no marks for

a perfect answer to a question. It is most unlikely that any examiner nowadays would be so strict; a common procedure is to mark all the answers presented, and choose the ones bearing the best marks. But even so, answering more than you are told means that some of your work will not gain you marks. If you have answered the required number and have time to spare, use it in checking the answers you have done. There have been cases of examinees who have answered their questions perfectly, and have so much time to spare that they have answered another question, just to ensure that they score 100 per cent. But such people are rare, and do not require the sort of advice given here.

Examiners, Papers and Questions

Remember that most examiners are reasonable people, and sometimes quite nice people. They are not always trying to catch you out. Boards of examiners do not like failing large numbers of students. Neither do they agree beforehand to fail a certain percentage of the candidates. This is a very common erroneous belief among students: that, say the fifteen poorest in a class of sixty are bound to fail, because this is the agreed policy. I can only state, firstly, that no board of which I have been a member has ever adopted this policy, and secondly, that so far as I know, no other board has. If fifteen out of sixty candidates fail, it is because they do not achieve the required pass standard, and not because they are the poorest fifteen of the bunch.

Of course, there have been, and there are, spiteful examiners. There is a story, which I am sure is untrue, of an examiner who, while setting a paper at his desk, leaned back with a satisfied smile and said to his colleague, 'Ah! They'll never do that one!' But even spiteful examiners have their papers scrutinised by moderators, to try to ensure that a paper is fair. Also, a degree examination has two functions. The first and obvious one is to find which students have learned enough to merit their progress to the next stage of the course. The second and less obvious one is that of grading. A degree examination is not a simple ability test such as you may find in training programmes for the armed forces, where you are taught a simple skill and have simply to show that you have learned it. The required pass mark in such a test may be from 70 per cent to as high as 90 per cent. I have known students with this 'ability test' mentality protest quite energetically because an examination question was not an almost exact copy of tutorial questions. But a high academic qualification is not earned by just learning to solve a range of routine problems. A good examination question is one which compels the examinee to think, and to work out how what has been learned on the course is to be applied to obtain a solution. In this way, the good candidate is distinguished from the mediocre or the poor one.

Finally, it must be confessed that luck may play an important part. The hard-working conscientious student who has worked steadily throughout the year may have an off-day, combined with a tough paper and score a mere 55 per cent, while a less industrious friend who began to work seriously only two weeks previously happened to 'mug up' two or three topics which turned up in the paper, and collects 60 per cent. There is no answer to this example of life's unfairness, except to wish you good luck!

How to Use the Book

First of all, this is a revision book, not a reference textbook. It is intended for use when you have completed, or nearly completed a first-year course in engineering mechanics and are about to prepare yourself for an examination. You should by now have a set of notes, either taken by yourself during the lectures, or in the form of hand-outs from lecturers, or received via a correspondence course. You may also have one or more textbooks. If you are taking a degree course, you have for some time been forcibly fed with concentrated doses of material in this and several other subjects, and in addition have probably been given sheets of examples which you have been advised to work through but just haven't found the time to do so. But the examination is now uncomfortably close; it is time to begin revising.

Here is a suggested procedure.

1. Choose a topic for revision.
2. Read up your notes on this topic and be sure you understand at least a good proportion of them reasonably well.
3. Look up the chapter in this book covering the topic. Each chapter begins with a Fact Sheet, which is a sheet of formulae and definitions covering the topic. Be sure that you can understand the contents of the Fact Sheet. You may encounter difficulties here. For example:
 - (a) The Fact Sheet may include formulae or definitions which you do not have in your notes. For instance, the Fact Sheet may define moment of inertia, whereas your notes might assume, perhaps incorrectly, that you understood this before beginning the course.
 - (b) Depending on your course, the course coverage of a topic may be wider and deeper than in this book.
 - (c) Symbols and notations may be unfamiliar. If so you must try to be flexible, and adjust. If you can solve the equation

$$x^2 - 9x - 30 = 0$$

but cannot solve

$$h^2 - 9h - 30 = 0$$

without first writing 'Let $h = x$ ' you are up against a difficulty which only you can resolve. If you are one of those people who *do* find this a major hang-up, then, as a part of your revision, write out the Fact Sheet yourself completely, transposing everything to your preferred system of notation.

4. Now look at one of the worked examples. Do not just read it through. Begin each example by trying to solve it yourself. So have paper, pen and calculator to hand. Attempt a solution, and only when you come up against a difficulty refer to the book, to find out what you have forgotten, or where you have gone wrong. The book solutions include helpful comments and explanations, in contrasting type: these, of course, would not be required in an examination

solution. When a calculation uses material from the Fact Sheet this is usually indicated by quoting the appropriate key letter in brackets at the beginning of the line.

5. Having worked successfully through an example, look through your own example sheets your textbooks, your old examination papers and the un-worked problems at the end of the chapter, and select a problem you think you could solve. At this stage, select only those problems for which you have answers. If you fail to get the right answer then try to find out where the mistake is, but don't spend too much time doing this. It may be only a small slip which in an examination would cost you only a mark or two. But later, get someone to check your work for you, if you can. The mistake may be more serious, and in any case you need to know what it is.

Hopefully, by following this procedure, you should be able to achieve a fair degree of competence in answering questions in your chosen topic, and move to another. But do not make the mistake of supposing that possession of a book of worked examples releases you from the necessity of thoroughly learning your subject. On the contrary, one of the aims of this book is to help you to learn your subject. Used properly, it should reinforce and extend your knowledge of dynamics. But it is not intended, and should not be used, as a substitute for a textbook.

A word about the examples themselves. About a third of these are actual questions taken from the examination papers of various institutions. The rest are designed specifically for this book. Although an attempt has been made to provide questions of approximately examination standard, some of the questions are below examination standard and some above it. (In Chapter 1, the nature of the topic has dictated the use of examples which take too long to answer to form satisfactory examination questions.) For this reason, you should not necessarily be discouraged if you find that you require more time to obtain a solution than you would be allowed in an examination. On the other hand, of course, neither should you become complacent if you solve a question in much less than 'examination' time.