Advanced mathematics

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Preface

This is the first of a series of books written for students preparing for A level Mathematics. Books 1 and 2 of the series cover the work required for a single-subject A level in Mathematics or in Pure and Applied Mathematics. Book 1 covers the essential core of sixth-form mathematics now accepted by the GCE Boards, while Book 2 covers the applied mathematics, i.e. the numerical methods, mechanics and probability, contained in most single-subject syllabuses. Book 3 covers the additional pure mathematics needed by students taking the double-subject Mathematics and Further Mathematics, and by those taking Pure Mathematics as a single subject.

Vector notation and vector techniques are used wherever appropriate, particularly where these methods illuminate or simplify the work, but their use is avoided whenever they appear likely to confuse the student. Set language is employed wherever it is considered helpful, but it is not introduced at all times as a matter of principle.

The material is arranged under well-known headings and is organised so that the teacher is free to follow his or her own preferred order of treatment. The chapter contents are listed and an index is also provided to make it easy for both the teacher and the student to refer back rapidly to any particular topic. For ease of reference, a list of the notation used is given at the back of the book together with a list of formulae.

Each topic is developed mainly through worked examples. There is a brief introduction to each new piece of work followed by worked examples and numerous simple exercises to build up the student's technical skills and to reinforce his understanding. It is hoped that this will enable the individual student working on his own to make effective use of the books, and the teacher to use them with mixed ability groups. At the end of each chapter there are many miscellaneous examples, taken largely from past A level examination papers. In addition to their value as examination preparation, these miscellaneous examples are intended to give the student the opportunity to apply the techniques acquired from the exercises throughout the chapter to a considerable range of problems of the appropriate standard.

We are most grateful to the University of London University Entrance and School Examinations Council (L), the Associated Examining Board (AEB), the University of Cambridge Local Examinations Syndicate (C) and the Joint Matriculation Board (JMB) for giving us permission to use questions from their past examination papers.

We are also grateful to the staff of Macmillan, particularly Mrs Janet Hawkins and Mr Tony Feldman, for the patience they have shown and the help they have given us in the preparation of these books.

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