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Operational Research

T. A. Burley
and
G. O'Sullivan

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Preface

Our purpose in writing this book is to convey an understanding of the methods of operational research. It has been developed out of many years of teaching this subject on professional-stage accountancy courses and second-year degree and B/TEC higher national courses. We have sought to provide a significant descriptive input in addition to numerical and algebraic aspects.

The book assumes an algebraic facility of about 'O' level standard. It also assumes a familiarity with such ideas of probability and statistics as are covered in the foundation stage of professional accountancy courses or in the first-year 'quantitative methods' units on degrees in business studies, accountancy and economics or on B/TEC higher national courses in the business area. The use of an electronic calculator will be found helpful for a number of the exercises.

The first chapter provides an overview of the subject of operational research and an introduction to model building. This is followed by a further twelve chapters each of which is based on a particular area of operational research. Each chapter has a flowchart detailing the links between the topics, and an introductory section which is followed by a progression of examples and exercises. Full answers to the examples are given in the text. Where student exercises require numerical solutions these are given in Appendix 1. Solutions are not given for exercises requiring descriptions, tables and diagrams. We have found this arrangement of material to be the basis of a popular and successful teaching method over a number of years.

In order to provide a 'rich feast' of examples and exercises we have referred to a number of past examination questions set by ACCA and ICMA. The permission of these bodies to use their questions is gratefully acknowledged elsewhere. We believe that the book will be very useful to students studying for the professional-stage examinations of these bodies as well as to second-year students on degree courses in business studies, accountancy and economics or on B/TEC higher national courses in the business area.

After the attempt in Chapter 1 to explain the scope of operational research and set the subject in its historical and practical context, the remainder of the book is largely devoted to consideration of particular OR models. This is to say we concentrate on the modelling of fairly simple situations, solving the models and interpreting the results. We hope that the reader in working through the book will not get lost in the 'trees' of detail of particular techniques and lose consciousness of the operational research 'wood' of which they are part. However, we believe that a thorough understanding of these techniques is a major objective of most people who will use this book. We also believe that this is the aspect of operational research most suited to textbook treatment. Issues of definition and data collection for substantial problems are better dealt with through project work, and the political and administrative issues associated with implementation can be fully appreciated only through experience in the practical situation.

In Chapters 2, 3 and 4 we look at models based on the ideas of linear programming. Chapter 2 treats linear programming explicitly from both graphical and algebraic viewpoints. Chapter 3 deals with the transportation problem, which is a particular kind of linear programming problem for which special methods of solution are available. Then Chapter 4 looks at the theory of games, for which the

general problem can be solved using the standard linear programming methods of Chapter 2.

Chapter 5 looks at networks as a method for scheduling complex projects involving a number of activities. Different types of network diagram are considered, as well as aspects such as manpower deployment, and in the last section probabilities are used to deal with projects where activity durations are not known with certainty.

Chapter 6 considers stock control and is a particularly good example of the modelling process at work. We begin by considering a model which is very simple indeed and look at the solutions it offers. We then move to models which offer progressively closer approximation to reality, including the use of probabilities to take account of variable demand.

Chapter 7 looks at queuing models, beginning with a very simple model for which features such as average queue-length and average waiting-time are calculated.

Results for slightly more complex models are then quoted and used in the solution of problems.

Chapter 8 concerns simulation and the example on queuing simulation when contrasted with the approach in Chapter 7 provides a good illustration of the distinction between mathematical and simulation models.

Chapter 9 introduces dynamic programming. This is an approach to problems involving sequences of decisions or other activities by working *recursively*. That is to say, we start by looking at the final subsystem and work back from a solution to that to a solution for the system as a whole.

Chapter 10 considers replacement problems, looking at how decisions on scale and frequency of replacement are taken in the light of probabilities derived from past experience.

Chapters 11 and 12 look at major applications of matrix algebra in input-output analysis and in Markov processes.

Finally, Chapter 13 considers forecasting methods, beginning with a simple forecasting model and progressing as far as multiple regression models.

We hope that the reader will be helped to gain a clearer insight into what is covered in the remaining units by consideration of the introduction in Chapter 1 to the philosophy and approach of operational research, the variety of its techniques and the versatility of its applications.

*Department of Mathematics and Statistics,
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T. A. B.
G. O'S.

Acknowledgements

We acknowledge with thanks the permission given by the following professional bodies to reproduce past examination questions set by them:

The Association of Certified Accountants (ACCA);

The Institute of Cost and Management Accountants (ICMA).

Where such questions are used as examples and exercises they are acknowledged by a statement at the end of the question of the initials of the body concerned and of the part of the examination in which they appeared. All answers to examples and solutions to exercises are completely our own and do not depend in any way on model answers produced by or on behalf of the professional bodies.

We also acknowledge the help of colleagues and students of the City of Birmingham Polytechnic who have helped us in the development and refining of the material presented in this book.