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Spatial Structure and the Microcomputer

Selected Mathematical Techniques

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Contents

<i>Table of computer programs</i>	<i>viii</i>
<i>Preface</i>	<i>ix</i>
1 Introduction	1
1.1 The computer	2
1.2 The equipment	4
1.3 Software	4
1.4 Functions, procedures and sub-routines	5
1.5 Errors, mistakes, blunders and inaccuracies	6
2 Mathematical processes for computation	9
2.1 Integration	9
2.2 Derivatives	11
2.3 Complex numbers	11
2.4 Vectors	14
3 Matrices	17
3.1 Determinants	19
3.2 Rank	19
3.3 Multiplication of matrices	20
3.4 Inversion	20
3.5 Least squares	22
3.6 The generalised inverse	23
3.7 Eigenvalues and vectors	24
3.8 The solution of the cubic equation and the diagonalisation of a 3×3 real symmetric matrix	26
3.9 To solve N non-linear equations $F_i(x_j) = 0$	27
3.10 Computation as a substitute for algebra	28
4 Coordinate systems	29
4.1 Cartesian coordinates	29
4.2 Cylindrical and spherical coordinates	30
4.3 Area and volume in Cartesian coordinates	31
4.4 Distances, bond angles and torsion angles	32

4.5	Lines and planes	36
4.6	To draw an ellipse	36
4.7	The helix	37
4.8	The ellipsoid	37
4.9	Rotations	37
4.10	Crystallographic and orthonormal axes	38
4.11	Networks	41
5	Geometry on the sphere: spherical trigonometry	46
5.1	Napier's Rules	48
5.2	The area of a triangle	48
5.3	Stereographic projection	49
5.4	Mercator's projection	51
5.5	The Hammer-Aitoff projection	52
5.6	Optimisation	52
5.7	To arrange points at random on a sphere	54
6	APL – a higher level language	56
6.1	To fit a polynomial to a curve	57
6.2	To fit a polynomial to a surface	58
6.3	Curve fitting	58
6.4	Splines	58
6.5	To fit a polynomial	59
7	The partition of space into domains	62
7.1	Voronoi polygons (or polyhedra)	62
7.2	Methods of calculation	63
7.3	Statistical methods	64
7.4	Exact calculation	65
8	The best fit between two shapes or molecules	67
8.1	Quaternions	68
8.2	The transformation of non-orthogonal axes (in N dimensions) to orthogonal axes. The Gram-Schmidt procedure	71
9	Pattern generation	76
9.1	The solar system	76
9.2	Fibonacci patterns	78
9.3	The Penrose pattern	80
9.4	The Julesz pattern – a test for stereo perception	80
10	Fourier transforms	83
10.1	Introduction to the Fourier transform	88
10.2	The discrete Fourier transform	90

<i>Contents</i>	<i>vii</i>
10.3 Computing the one-dimensional Fourier transform	91
10.4 Sampling and resolution	95
10.5 The fast Fourier transform	96
10.6 Separability (the two-dimensional transform of a square array)	103
10.7 The inverse FFT	106
10.8 Computing the transform of real arrays	107
11 Applications and other transformations	109
11.1 Convolution	109
11.2 Correlation	116
11.3 Other transformations	120
11.4 Walsh transform	124
11.5 Hadamard transform	128
<i>Appendix: Computer programs for numerical and geometrical analysis</i>	<i>132</i>
<i>References</i>	<i>199</i>
<i>Further reading</i>	<i>200</i>
<i>Index</i>	<i>202</i>

Table of computer programs

BASIC programs

BINARY	132
CHOLESKI	133
CONVFT	135
DETER	137
DISC	138
FFT	140
FIBONACCI	143
FWT	143
GENDAT	145
GENINV	147
GREEK	150
HADAMARD	153
IMAGES	154
INVERT	159
JACOBI	160
JULESZ	164
LAPLACE	165
ONEDFT	166
PACK	167
PENROSE	170
PLOT	173
QUAT	178
RFT	181
SOLVE	183
TELLUR	184
TWODFT	188
VORON	190
WALSH	191

APL programs

DETER	193
GENINV	194
JACOBI	195
LANGLET	197

Preface

This book is intended for readers with interests in the representation and analysis of spatial structure using the microcomputer.

In almost every branch of science and engineering problems frequently arise which require solutions involving some aspect of spatial structure. For example, the graphical representation of complex three-dimensional surfaces and the improvement of two-dimensional pictorial data for human interpretation are just two problems requiring a combination of structural analysis and computer methods for successful solution. The following chapters present a variety of methods relevant to these and other topics which frequently arise in the course of solving problems involving spatial structure.

The earlier chapters present what is commonly known as the '*real*' space approach to spatial structure and involve the use of Cartesian systems to represent structures. The subsequent chapters are concerned with the representation of spatial data in '*transform*' space and describe methods frequently used in solving problems which arise in pattern recognition and signal processing.

The level of mathematics presented should be within the grasp of most undergraduate students working in the fields of computing, engineering and the physical sciences. For those readers with less interest in the mathematics than in the practical application of the methods, the book should still appeal since many programs written in BASIC together with documentation and examples illustrating the running of the programs are provided in an appendix and can be used to apply the methods to different sets of data.

It is true to say that microcomputers have become a dominating force in the computing field. Calculations which several years ago would have taxed the resources of many of the larger computers can now be carried out by many of the commercially available microcomputers. Furthermore, with the development of the 16-bit and 32-bit microprocessors, the equivalent computing power is now available at a fraction of the original cost. Clearly, however, not all the problems encountered in problems of spatial structure can be solved using a microcomputer since in extreme cases the extent of the calculations together with the sheer volume of data involved prove unsolvable by even the largest computer yet invented. Nevertheless, many of the principles which involve the methods presented here can be usefully explored and in many cases used to provide solutions to problems of spatial structure.