Core Books in Advanced Mathematics

Methods of Algebra

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Methods of Algebra

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Preface

Advanced level mathematics syllabuses are once again undergoing changes in content and approach following the revolution in the early 1960s which led to the unfortunate dichotomy between ‘modern’ and ‘traditional’ mathematics. The current trend in syllabuses for Advanced level mathematics now being developed and published by many GCE Boards is towards an integrated approach, taking the best of the topics and approaches of modern and traditional mathematics, in an attempt to create a realistic examination target through syllabuses which are maximal for examining and minimal for teaching. In addition, resulting from a number of initiatives, core syllabuses are being developed for Advanced level mathematics consisting of techniques of pure mathematics as taught in schools and colleges at this level.

The concept of a core can be used in several ways, one of which is mentioned above, namely the idea of a core syllabus to which options such as theoretical mechanics, further pure mathematics and statistics can be added. The books in this series are core books involving a different use of the core idea. They are books on a range of topics, each of which is central to the study of Advanced level mathematics, which together cover the main areas of any single-subject mathematics syllabus at Advanced level.

Particularly at times when economic conditions make the problems of acquiring comprehensive textbooks giving complete syllabus coverage acute, schools and colleges and individual students can collect as many of the core books as they need to supplement books they already have, so that the most recent syllabuses of, for example, the London, Cambridge, AEB and JMB GCE Boards can be covered at minimum expense. Alternatively, of course, the whole set of core books gives complete syllabus coverage of single-subject Advanced level mathematics syllabuses.

The aim of each book is to develop a major topic of the single-subject syllabuses, giving essential book work, worked examples and numerous exercises arising from the authors’ vast experience of examining at this level. Thus, the core books, as well as being suitable for use in either of the above ways, are ideal for supplementing comprehensive textbooks by providing more examples and exercises, so necessary for the preparation and revision for examinations.

The ability to carry out basic algebraic manipulations accurately and quickly is essential for Advanced level mathematics and is the key to success in many other aspects of mathematics. In this particular book we cover the

algebraic techniques essential for the core syllabus of pure mathematics now being included by GCE Examining Boards in Advanced level syllabuses. The many worked examples illustrating the various techniques employed form an essential part of this book and are intended to ensure that the conscientious student acquires a mastery of manipulative processes involving functions (algebraic and transcendental), indices, surds, polynomials, quadratic functions and equations, sequences and series, and inequalities. In addition, a chapter on mathematical proof enumerates the different types of mathematical proof expected to be known at this level and contains many algebraic illustrations.

The examples and exercises throughout the book are illustrative of those now being set in single-subject Advanced level mathematics by the GCE examining boards.

John E. Hebborn
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