Core Books in Advanced Mathematics

Proof

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Preface

Advanced level mathematics syllabuses are once again undergoing changes of content and approach, following the revolution in the early 1960s which led to the unfortunate dichotomy between 'modern' and 'traditional' mathematics. The current trend in syllabuses for Advanced level mathematics now being developed and published by many GCE boards is towards an integrated approach, taking the best of the topics and approaches of the modern and traditional, in an attempt to create a realistic examination target through syllabuses which are maximal for examining and minimal for teaching. In addition, resulting from a number of initiatives, core syllabuses are being developed for Advanced level mathematics consisting of techniques of pure mathematics as taught in schools and colleges at this level.

The concept of a core can be used in several ways, one of which is mentioned above, namely the idea of a core syllabus to which the options, such as theoretical mechanics, further pure mathematics and statistics can be added. The books in this series are core books involving a different use of the core idea. They are books on a range of topics each of which is central to the study of Advanced level mathematics. The books also form small core studies of their own, of topics which, together, cover the main areas of any single-subject mathematics syllabus at Advanced level.

The series will be found particularly valuable at times when economic conditions create acute problems in the acquisition of comprehensive textbooks giving complete syllabus coverage. Schools, colleges and individual students can collect as many of the core books as they need to supplement books they already possess, so that the most recent syllabuses of, for example, the London, Cambridge, AEB and JMB GCE Boards, can be covered at minimum expense. Alternatively, of course, the whole set of core books gives complete syllabus coverage of single-subject Advanced level mathematics.

The aim of each book is to develop a major topic of the single-subject syllabuses. All essential bookwork is given, together with worked examples and exercises arising from the authors' vast experience of examining at this level. Many questions of GCE Advanced level type are also included. Thus, the core books, as well as being used in the ways described above, would also be ideal for supplementing comprehensive textbooks by providing more examples and exercises, so necessary for preparation and revision for examinations on the Advanced level mathematics syllabuses offered by the GCE Boards.

The subject of this volume of the series is mathematical proof. During the

early years at secondary school, any sort of formal proof in mathematics is no longer required of pupils. Gone are the days of the geometry theorem carefully written down and learnt (often by heart!). Instead, the time is spent on familiarising the pupils with new concepts and on the acquisition of skills in dealing with those concepts. However, at the later stages of a mathematics course and certainly at sixth-form level, the formal proof, that is the carefully reasoned argument, is necessary and it is not always realised what difficulties this presents to students. Whilst textbooks introduce proofs of various kinds at this level, very little is said about the nature of the process and of the various forms that it can take. Thus, what is often seen as a proof by students is a sequence of statements, often relevant, but lacking the necessary connectives to constitute a proof. When, therefore, it comes to a particular form of proof, for example mathematical induction, where the correct reasoning is absolutely crucial to the argument, students tend to find great difficulty. All too often, examiners find that perfectly correct statements are put together invalidly to give a 'proof' that does not even make sense.

The present book has been written in an attempt to help sixth-form teachers and students by discussing the nature of proof and by setting out the various forms of proof which are available. Experience shows that first-year university students could also profit from a study of this book. Many proofs are given in full for the guidance of students and there are numerous examples for them to work. These include many of the more sophisticated types of multiple-choice items which are found in some Advanced level mathematics papers nowadays. It is hoped, therefore, that the book will be valuable to many university students and to all A-level students and their teachers.

Examples in the book which are suitable for candidates doing double-subject mathematics or pure mathematics only are marked with an asterisk and those suitable for candidates doing applied mathematics are marked with two asterisks.

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