Advanced mathematics

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C W Celia

*formerly Principal Lecturer in Mathematics,
City of London Polytechnic*

A T F Nice

*Mathematics Department, Lady Eleanor Holles
School, Hampton; formerly Principal Lecturer
in Mathematics, Middlesex Polytechnic*

K F Elliott

*formerly Head of the Division of Mathematics
Education, Derby Lonsdale College of Higher
Education*

*Consultant Editor: Dr C. Plumpton, Moderator
in Mathematics, University of London School
Examinations Board; formerly Reader in
Engineering Mathematics, Queen Mary
College, London*

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First edition 1985

Reprinted 1986, 1991

Published by
MACMILLAN EDUCATION LTD
Houndmills, Basingstoke, Hampshire RG21 2XS
and London
Companies and representatives
throughout the world

British Library Cataloguing in Publication Data
Celia, C.W.

Advanced mathematics.

3

I. Mathematics—1961—

I. Title II. Nice, A. T. F.

III. Elliott, K. F.

510 QA39.2

ISBN 978-0-333-34827-7

ISBN 978-1-349-06711-4 (eBook)

DOI 10.1007/978-1-349-06711-4

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Preface

This is the third of a series of books written for students preparing for A level Mathematics. Book 1 covers the essential core of sixth-form mathematics now accepted by the GCE Boards, while Book 2 covers the applied mathematics, i.e. the numerical methods, mechanics and probability, contained in most single-subject mathematics syllabuses.

Book 3 covers the additional pure mathematics needed by students taking the double subject Mathematics and Further Mathematics, and by those taking Pure Mathematics as a single subject.

It must be emphasised that the reader is expected to be familiar with the use of a calculator, so that numerical work presents no difficulties.

The material is arranged under well-known headings and is organised so that the teacher is free to follow his or her own preferred order of treatment. The chapter contents are listed and an index is also provided to make it easy for both the teacher and the student to refer back rapidly to any particular topic. For ease of reference, a list of the notation used is given at the back of the book together with a list of formulae.

The approach in Book 3 is the same as in Books 1 and 2, each topic being developed mainly through worked examples. There is a brief introduction to each new piece of work followed by worked examples and numerous simple exercises to build up the student's technical skills and to reinforce his or her understanding. It is hoped that this approach will enable the individual student working on his or her own to make effective use of the books and the teacher to use them with mixed ability groups. At the end of each chapter there are many miscellaneous examples, taken largely from past A level examination papers. In addition to their value as examination preparation, these miscellaneous examples are intended to give the student the opportunity to apply the techniques acquired from the exercises throughout the chapter to a considerable range of problems of the appropriate standard.

We are most grateful to the University of London University School Examinations Board (L), the Associated Examining Board (AEB), the University of Cambridge Local Examination Syndicate (C) and the Joint Matriculation Board (JMB) for giving us permission to use questions from their past examination papers.

We are also grateful to the staff of Macmillan Education for the patience they have shown and the help they have given us in the preparation of these books.

C. W. Celia

A. T. F. Nice

K. F. Elliott