Core Books in Advanced Mathematics

Differentiation

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Differentiation

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Preface

Advanced level mathematics syllabuses are once again undergoing changes of content and approach, following the revolution in the early 1960s which led to the unfortunate dichotomy between 'modern' and 'traditional' mathematics. The current trend in syllabuses for Advanced level mathematics now being developed and published by many GCE Boards is towards an integrated approach, taking the best of the topics and approaches of the modern and traditional, in an attempt to create a realistic examination target, through syllabuses which are maximal for examining and minimal for teaching. In addition, resulting from a number of initiatives, core syllabuses are being developed for Advanced level mathematics syllabuses, consisting of techniques of pure mathematics as taught in schools and colleges at this level.

The concept of a core can be used in several ways, one of which is mentioned above, namely the idea of a core syllabus to which options such as theoretical mechanics, further pure mathematics and statistics can be added. The books in this series are core books involving a different use of the core idea. They are books on a range of topics, each of which is central to the study of Advanced level mathematics; they form small core studies of their own, of topics which together cover the main areas of any single-subject mathematics syllabus at Advanced level.

Particularly at times when economic conditions make the problems of acquiring comprehensive textbooks giving complete syllabus coverage acute, schools and colleges and individual students can collect as many of the core books as they need, one or more, to supplement books already possessed, so that the most recent syllabuses of, for example, the London, Cambridge, AEB and JMB GCE Boards, can be covered at minimum expense. Alternatively, of course, the whole set of core books give complete syllabus coverage of single-subject Advanced level mathematics syllabuses.

The aim of each book is to develop a major topic of the single-subject syllabuses, giving essential book work and worked examples and exercises arising from the authors' vast experience of examining at this level and including actual past GCE questions also. Thus, as well as using the core books in either of the above ways, they would also be ideal for supplementing comprehensive textbooks in the sense of providing more examples and exercises, so necessary for preparation and revision for examinations on the Advanced level mathematics syllabuses offered by the GCE Boards.

In this book on differentiation, it has been assumed that the reader has a

rudimentary knowledge of calculus. Consequently, whilst the basic results and general theorems on differentiation are given, no attempt has been made to prove them. The writers' main concern has been to try to ensure that, given basic results and general theorems, the readers can by (i) following the numerous worked examples and (ii) working out for themselves the frequent complementary exercises, be enabled to cope easily with all such examination questions to be found in the GCE A-level papers. It is also believed that many first-year university and polytechnic engineering and applied science students will find the text a most helpful aid in ensuring proficiency in differentiation and its applications. In an attempt to establish good techniques, the opportunity has been taken of drawing the reader's attention to quicker and better solutions whenever appropriate.

Approximately half of the book is devoted to the many applications of differential calculus.

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