Core Books in Advanced Mathematics

Integration

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Preface

Advanced level mathematics syllabuses are once again undergoing changes of content and approach, following the revolution in the early 1960s which led to the unfortunate dichotomy between 'modern' and 'traditional' mathematics. The current trend in syllabuses for Advanced level mathematics now being developed and published by many GCE Boards is towards an integrated approach, taking the best of the topics and approaches of the modern and traditional, in an attempt to create a realistic examination target, through syllabuses which are maximal for examining and minimal for teaching. In addition, resulting from a number of initiatives, core syllabuses are being developed for Advanced level mathematics syllabuses, consisting of techniques of pure mathematics as taught in schools and colleges at this level.

The concept of a core can be used in several ways, one of which is mentioned above, namely the idea of a core syllabus to which options such as theoretical mechanics, further pure mathematics and statistics can be added. The books in this series are core books involving a different use of the core idea. They are books on a range of topics, each of which is central to the study of Advanced level mathematics; they form small core studies of their own, of topics which together cover the main areas of any single-subject mathematics syllabus at Advanced level.

Particularly at times when economic conditions make the problems of acquiring comprehensive textbooks giving complete syllabus coverage acute, schools and colleges and individual students can collect as many of the core books as they need, one or more, to supplement books already possessed, so that the most recent syllabuses of, for example, the London, Cambridge, AEB and JMB GCE Boards, can be covered at minimum expense. Alternatively, of course, the whole set of core books give complete syllabus coverage of single-subject Advanced level mathematics syllabuses.

The aim of each book is to develop a major topic of the single-subject syllabuses, giving essential book work and worked examples and exercises arising from the authors' vast experience of examining at this level and including actual past GCE questions also. Thus, as well as using the core books in either of the above ways, they would also be ideal for supplementing comprehensive textbooks in the sense of providing more examples and exercises, so necessary for preparation and revision for examinations on the Advanced level mathematics syllabuses offered by the GCE Boards.

This book on integration follows closely the ideas developed in the companion

book on differentiation, and is intended to be read in conjunction with that book.

Integration is treated as the reverse process to differentiation and, as such, the basic results are tabulated. From them a more generalised table of results is derived and the first six chapters are devoted to ensuring that the reader becomes familiar with the accepted techniques of integration – i.e. integration by substitution, the involvement of trigonometric functions, rational and irrational algebraic fractions and integration by parts. Definite integrals and the properties of definite integrals are also discussed and, provided the readers follow carefully the many worked examples and try for themselves the complementary exercises, they should experience little difficulty in coping with the integrals to be found in most GCE single-mathematics examination papers. The authors also believe that many first-year university and polytechnic engineering and applied science students will find the book a valuable aid to ensuring proficiency in integration.

Applications of integration, approximate methods of integration and simple first-order variable separable type differential equations are also discussed in the second part of the book.

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