Core Books in Advanced Mathematics

Mechanics of Groups of Particles

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Preface

Advanced level mathematics syllabuses are once again undergoing changes in content and approach following the revolution in the early 1960s which led to the unfortunate dichotomy between ‘modern’ and ‘traditional’ mathematics. The current trend in syllabuses for Advanced level mathematics now being developed and published by many GCE Boards is towards an integrated approach, taking the best of the topics and approaches of modern and traditional mathematics, in an attempt to create a realistic examination target through syllabuses which are maximal for examining and minimal for teaching. In addition, resulting from a number of initiatives, core syllabuses are being developed for Advanced level mathematics consisting of techniques of pure mathematics as taught in schools and colleges at this level.

The concept of a core can be used in several ways, one of which is mentioned above, namely the idea of a core syllabus to which options such as theoretical mechanics, further pure mathematics and statistics can be added. The books in this series involve a different use of the core idea. They are books on a range of topics, each of which is central to the study of Advanced level mathematics; they form small core studies of their own, of topics which together cover the main areas of any single-subject mathematics syllabus at Advanced level.

Particularly at times when economic conditions make the problems of acquiring comprehensive textbooks giving complete syllabus coverage acute, schools and colleges and individual students can collect as many of the core books as they need to supplement books they already have, so that the most recent syllabuses of, for example, the London, Cambridge, AEB and JMB GCE Boards can be covered at minimum expense. Alternatively, of course, the whole set of core books gives complete syllabus coverage of single-subject Advanced level mathematics syllabuses.

The aim of each book is to develop a major topic of the single-subject syllabuses giving essential book work, worked examples and numerous exercises arising from the authors’ vast experience of examining at this level. Thus, as well as using the core books in either of the above ways, they are ideal for supplementing comprehensive textbooks by providing more examples and exercises, so necessary for the preparation and revision for examinations.

A feature of the books in this series is the large number of worked examples which are regarded as an integral part of the text. It is hoped that these books will be of use not only to schools but also to the large minority of candidates who are studying by themselves. Mechanics, perhaps more so than other

branches of mathematics, can be mastered only by working conscientiously and thoughtfully through many worked and unworked examples. This is the only way in which the essential simplicity of the subject can be appreciated.

In order to achieve a degree of uniformity in the size of the books in this series, the Advanced level mechanics topics are covered in two books. The first book, *Newton's Laws and Particle Motion*, considered the basic principles of mechanics and applied them to particle motion. In this book basic kinematics and Newton's laws, as covered in the first book, are used to discuss energy, groups of particles, statics and rigid-body motion.

The treatment in both books assumes a knowledge of vector algebra and of elementary calculus (including the exponential and logarithm functions). These subjects are covered in other volumes in the series. The worked examples and problems have been carefully chosen to be physically important (as distinct from artificial examples devoid of any relevance to the real world). The treatment unashamedly makes appeals to intuition and common-sense and the ordering of the topics (sometimes unconventional) has been arranged to stress that all mechanics is based on Newton's laws, not on a series of *ad hoc* techniques. One final feature is the stress on dimensional consistency and the possibility of checking that this allows.

Whilst the notation used is generally self-explanatory, it should be noted that there are two distinct ways of specifying the inverse trigonometric functions. Thus $\tan^{-1} x$ and $\arctan x$ are both used to denote the angle θ between $-\pi/2$ and $\pi/2$ such that $\tan \theta = x$. Also, the symbol $\approx$ means 'approximately equals'.

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