Core Books in Advanced Mathematics

Vectors

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General Editor: C. PLUMPTON, Moderator in Mathematics,
University of London School Examinations Department;
formerly Reader in Engineering Mathematics,
Queen Mary College, University of London.

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Tony Bridgeman

Chief Examiner in Advanced Level Mathematics,
University of London School Examinations
Department, Senior Lecturer in Applied
Mathematics, University of Liverpool.

P. C. Chatwin

Assessor in Advanced Level Mathematics,
University of London School Examinations
Department, Senior Lecturer in Applied
Mathematics, University of Liverpool.

C. Plumpton

Moderator in Mathematics, University of
London School Examinations Department;
formerly Reader in Engineering Mathematics,
Queen Mary College, University of London.

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Preface

Advanced level mathematics syllabuses are once again undergoing changes of content and approach, following the revolution in the early 1960s which led to the unfortunate dichotomy between ‘modern’ and ‘traditional’ mathematics. The current trend in syllabuses for Advanced level mathematics now being developed and published by many GCE Boards is towards an integrated approach, taking the best of the topics and approaches of the modern and traditional, in an attempt to create a realistic examination target, through syllabuses which are maximal for examining and minimal for teaching. In addition, resulting from a number of initiatives, core syllabuses are being developed for Advanced level mathematics syllabuses, consisting of techniques of pure mathematics as taught in schools and colleges at this level.

The concept of a core can be used in several ways, one of which is mentioned above, namely the idea of a core syllabus to which options such as theoretical mechanics, further pure mathematics and statistics can be added. The books in this series are core books involving a different use of the core idea. They are books on a range of topics, each of which is central to the study of Advanced level mathematics; they form small core studies of their own, of topics which together cover the main areas of any single-subject mathematics syllabus at Advanced level.

Particularly at times when economic conditions make the problems of acquiring comprehensive textbooks giving complete syllabus coverage acute, schools and colleges and individual students can collect as many of the core books as they need, one or more, to supplement books already acquired, so that the most recent syllabus of, for example, the London, Cambridge, JMB and AEB GCE Boards, can be covered at minimum expense. Alternatively, of course, the whole set of core books gives complete syllabus coverage of single-subject Advanced level mathematics syllabuses.

The aim of each book is to develop a major topic of the single-subject syllabuses, giving essential book work and worked examples and exercises arising from the authors’ vast experience of examining at this level and including actual past GCE questions also. Thus, as well as using the core books in either of the above ways, they would also be ideal for supplementing comprehensive textbooks in the sense of providing more examples and exercises, so necessary for preparation and revision for examinations on the Advanced level mathematics syllabuses offered by the GCE Boards.

Vectors are now regarded as an essential tool in mathematics, physics and

engineering and the purpose of this book is to provide a systematic and coherent introduction to the subject of vector algebra. The many worked examples illustrate the elegance and practical convenience of vector methods in physical and geometrical problems. As with other branches of mathematics, vector algebra can only be mastered, and fully appreciated, by working conscientiously through the worked and unworked examples. The treatment assumes an elementary knowledge of calculus, a topic which is covered by companion volumes in this series.

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Tony Bridgeman
P. C. Chatwin
C. Plumpton
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