ENERGY METHODS OF STRUCTURAL ANALYSIS

By the same author

ESSENTIAL SOLID MECHANICS
Theory, Worked Examples and Problems

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Theory, worked examples and problems

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PREFACE

A wide variety of different methods of structural analysis exist although many of them are designed for the solution of particular types of problem. Two procedures, however, are generally applicable; these are the method of virtual work and energy methods. In essence, the two methods are equivalent since, although the arguments used in establishing the governing equations differ, the equations themselves are identical.

In the author's experience, students studying virtual work find some difficulty in coming to terms with the idea of hypothetical forces or deformations whereas they quickly grasp the more obvious physical interpretations of the energy approach. One additional advantage that energy methods have over virtual work is their application to the approximate solution of complex problems for which exact solutions may not exist.

Energy methods have not received the consideration they deserve in the literature and it is for this reason that this book has been written. The few acceptable textbooks which treat the subject are at a level which is not easily accessible to undergraduates. It is a matter of regret that most undergraduate textbooks which do deal with energy methods generally reveal a hazy understanding of the principles and fail to take full advantage of their potential.

Energy methods have applications in almost every branch of structural analysis, as well as in many other fields. It is therefore essential that the undergraduate engineer should be fully conversant with their use. At the same time, it must be remembered that there are certain alternative specialised methods of analysis which might be quicker or easier for the solution of particular problems; it is the reader's responsibility to ensure that he does not put all his eggs in one basket.

This book is intended for second and third-year undergraduates in university or polytechnic degree courses. It should also prove useful as a source of reference to designers in practice. The format follows that of the author's previous book in this series, *Essential Solid Mechanics*; the theory, in concise form, is followed by a number of worked examples chosen to illustrate all the principles involved. Each chapter (except the first) ends with a selection of problems with answers which the reader may use for practice. The examples and problems are typical of those set in examinations at the end of the second or third years of a degree course. In some cases the precise origin of the questions is unknown but general acknowledgement is given here. The author alone is responsible for the solutions and answers.

One feature which will be of particular interest to both final-year undergraduates and practising designers is the design example which completes each of the three main chapters. Here an attempt

has been made to create real-life design problems which may be solved by the application of energy methods. These examples must not be confused with the examination type of question.

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