Digital Circuits for Binary Arithmetic

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**No technical science
can exist without
addition of numbers**

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Preface

Since the publication of the first book on electronic switching circuits by the staff of the Computation Laboratory of Harvard University in 1951 (*Synthesis of Electronic Computing and Control Circuits*, Harvard University Press, Massachusetts) many books on digital switching circuits and systems have been published, most of them starting with the fundamentals of switching algebra based on Boolean algebra (C. E. Shannon, *Symbolic Analysis of Relay and Switching Circuits*).

The field of application of digital techniques and systems has, since then, enormously increased in size and importance. The medium- and large-scale integrated circuits have introduced such a high degree of complexity and sophistication that many users of these switching elements only know how to apply them in accordance with the instructions given in the manufacturer's catalogue, but do not fully understand how they actually operate. Many users will never question the switching methods used in these sophisticated integrated circuits, nor will they criticise their application as long as a working system is obtained.

It requires a team of digital switching specialists and a handbook consisting of several volumes to discuss all the commercially available digital integrated circuits manufactured in various integration techniques. However, when limiting this field to TTL integrated circuits implementing one or other algorithm based on addition, and with that also subtraction, multiplication and division, it is possible to compile a moderately sized text on the design of 'Digital Circuits for Binary Arithmetic'. This means there is a limitation and, there are several reasons why this text cannot be complete. Firstly, so many ingenious switching proposals in which addition plays a part have been published in recent years that only the most important fundamental ideas could be incorporated in this introduction. Secondly, other bright ideas in this field will be published between the writing of this text and its publication.

The text of this book is not limited to the already published ideas about digital switching methods using addition algorithms and their commercially available implementations. It shows in many cases what is still missing or what alternative switching methods could also be considered. For a number of switching circuits the

economics of certain methods are considered and alternative, more economic, switching methods with existing integrated circuits are given. Cost improvements varying from 3 : 1 to about 10 : 1 can be obtained.

After discussing the fundamentals of the codes, the addition and subtraction of numbers in these codes are treated and considered from many points of view. This is a preparation for the discussion of the more complicated ideas behind the implementation of the multiplication and the division, as realised in a number of integrated circuits.

Applications of the fundamental methods are discussed in the chapters on BCD arithmetic, floating point arithmetic, accumulators and miscellaneous adding algorithms.

This book is not a summary of catalogue ideas and items; it shows a systematic and thoroughly argued development of digital switching ideas. It can be used as a textbook for teachers and graduate-level students. It might cause them to doubt the quality of the information sometimes printed elsewhere. Moreover, many scientific workers in laboratories and industry will find useful information in this introduction to additive digital switching that cannot be found in the manufacturers' catalogues.

The author thanks T. van Onzen for checking and testing many of the circuit ideas given in this text, H. A. Vink for the thoroughness of reading the text itself and to the members of his staff who have contributed their ideas. The author is responsible for drawing the diagrams and has paid much attention to it, as in many cases the form of these diagrams is instrumental to the explanation of their operation.

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R.M.M.O.