QUEUEING THEORY

Worked examples and problems

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PREFACE

The basic concepts and an understanding of modern queueing theory are requirements not only in the training of operational research staff, management scientists, etc., but also as fundamental concepts in the training of managers or in management development programmes.

The efficient design and operation of 'service functions' is one of the main problems facing management today and the understanding obtained from a study of queueing theory is essential in the solution of these problems.

Industry and commerce have for too long concentrated their main resources on designing and operating the 'production units' and little attention has been paid until recently to the 'service units'. Basic concepts such as 'increased efficiency is achieved when the utilisation of service units is reduced' are still hard for practical personnel to understand, brought up as they are on the concept of 'maximising the utilisation' of their facilities. The ancient Chinese civilisation had a system based on queueing theory: 'Pay your doctor only when you are well'. Thus in industry if a system is correctly designed, management should be happy when 'its maintenance gang is playing cards' since there are no breakdowns to be repaired!

This book, by concentrating on problems with their fully worked-out solutions, gives students of queueing theory not only a chance to test their understanding of the theory, but also illustrates the wide range of application of the theory.

The book covers the steady-state solutions of random-arrival queueing systems. It is designed to meet the needs not only of management science training programmes, but also of management teaching programmes.

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J. Murdoch

GLOSSARY OF SYMBOLS

D	deterministic distribution
G	general distribution
M	negative exponential distribution
M_n	negative exponential distribution with mean dependent on the number in the system n
C	number of channels
N	maximum system size in finite queues
λ	average arrival rate
$1/\lambda$	average inter-arrival time
μ	average service rate
$1/\mu$	average service time
$\rho = \frac{\lambda}{\mu}$ (or $\frac{\lambda}{C\mu}$)	intensity of traffic for single and multi-channel queues
or	
$\epsilon = \lambda/\mu$	traffic offering (multi-channel queues)
σ_s^2	variance of service time
$d(t)dt$	distribution of the time in the system (steady state)
n	number in the system
$\bar{n}$	average number in the system
$P_n(t)$	transient state probabilities of n in the system
P_n	steady-state probabilities of n in the system
q	number in the queue
$\bar{q}$	average number in the queue
$w(t)dt$	distribution of the waiting time in the queue in the steady state
$\bar{w}$	average waiting time of all customers in the queue in the steady state.

BASIC DISTRIBUTIONS

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \text{Poisson distribution (mean = m)}$$

$$P(t) = \frac{1}{T} e^{-t/T} dt$$

Negative exponential distribu-
tion (mean = T)

CLASSIFICATION OF QUEUEING SYSTEMS

Queues are classified in the book as follows.

(1)	/	(2)	/	(3)	/	(4)
(1)		<i>Input Distribution</i>		e.g.		M, D, M_n , G, etc.
(2)		<i>Service Distribution</i>		e.g.		G, D, M, etc.
(3)		<i>Number of Service Channels</i>		e.g.		1, 2, ... C, etc.
(4)		<i>Number in the System</i>		Unconstrained ∞ , finite,		maximum size = N

EXAMPLES OF USE OF CLASSIFICATION SYSTEM

Thus an M/M/1/ ∞ system is random arrival, negative exponential service time distribution, single-channel, no constraint on queue size.

Again a G/M/C/N system is general arrival distribution, negative exponential service distribution, C service channels, maximum number in the system N.

The service mechanism in all problems, is service in order of arrival, or first-in, first-out (FIFO) system.