Mechanical Science III

P. R. Lancaster

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Corresponding Section in TEC Standard Unit U75/058

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1. Uses stress, strain, elasticity in problems
 - Solves from first principles problems involving composite bars under uni-axial loads only, at uniform temperature.
 - Solves similar problems but including the effect of uniform temperature change.
 - States that total direct strain is the sum of the strain due to loading and temperature change.
 - Defines (a) shear stress, (b) shear strain and (c) modulus of rigidity (shear modulus) and solves associated problems.
 - Defines Poisson's ratio.
 - Applies the Poisson's ratio effect in stress-strain relationships to solve associated problems in two dimensions (excluding shear stress action).
2. Develops, and uses, the simple theory of bending of symmetrical beams
 - Identifies, with the aid of a sketch, the position of a neutral plane in a symmetrical beam under the influence of a bending force system.
 - Defines bending moment.
 - Shows, by use of the equations of equilibrium, that under equilibrium conditions (a) the neutral plane (axis) passes through the centroid of cross-section, (b) the bending stress is given by $\sigma = My/I$
 - Solves problems, with section characteristics restricted to rectangular, circular or idealised I-section beams, involving: (a) maximum allowable stresses, (b) bending moments or (c) loading conditions.
 - Given a specified section modulus (Z) uses standard section handbooks to select appropriate beams.

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3. Develops and uses the simple theory of torsion of circular-section bars

Describes the assumptions necessary to develop the simple theory of torsion of circular-section bars.

Derives from first principles the relationship between shear strain and twist per unit length, i.e.

$$\gamma = \frac{r\theta}{l} = \frac{\tau}{G}$$

Uses the equation of equilibrium to derive the further relationship

$$T = \frac{GJ\theta}{l}$$

Solves problems involving torsion in solid and hollow shafts.

B. DYNAMICS

- States the laws of, and solves problems on, angular motion
 - States the equations of motion, for angular motion with constant angular acceleration.
 - Solves problems involving constant angular acceleration.
 - Derives from first principles the relationship between applied torque, angular acceleration and moment of inertia.
 - Defines radius of gyration, k , by reference to the expression $I = mk^2$.
 - Solves problems relating to the angular motion of discs and flywheels.
 - Derives from first principles the expression $r\omega^2$ for centripetal acceleration of a body moving in a circle with uniform angular velocity.
 - Solves problems involving motion of bodies in a circle including considerations of banking.

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5. Describes, and solves problems involving, simple harmonic motion
 - Describes the relationships between (a) restoring force and displacement (b) displacement and time (c) velocity and time (d) acceleration and displacement.
 - Defines simple harmonic motion.
 - Derives the interrelationships between the quantities specified in (a), (b), (c) and (d) above.
 - Relates simple harmonic motion to circular motion of a phasor and circular frequency.
 - Solves problems involving simple harmonic motion including the simple pendulum and body supported by a spring.
 - Describes resonance as occurring when the applied frequency equals the natural frequency.
 - Discuss the problems that can arise when resonance occurs, e.g. in the use of tools.
6. Describes, and solves problems, involving linear and angular kinetic energy
 - Derives from first principles the expressions for (a) linear kinetic energy and (b) angular kinetic energy of a body.
 - Solves problems involving linear and/or angular kinetic energy, including flywheels and lift systems.

C. FLUIDS IN MOTION

7. States and uses Bernoulli's equation
States Bernoulli's equation.
States the equation of continuity for steady flow through tapered pipes.
Applies Bernoulli's equation to solve problems involving

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flow of liquids through pipes, including tapered and inclined pipes and orifices.

Uses the momentum principle to calculate the force produced by the impact of a liquid jet on a normal flat plate.

Preface

This book follows the Standard TEC Unit *Mechanical Science III* which is a *third-level* Unit in the certificate programmes in mechanical and production engineering.

The *specification* of the course is as follows.

Unit Title	Mechanical Science
Unit Level	III
Unit Value	One
Design Length	60 hours
Prerequisite Units	TEC U75/036 Engineering Science II
Credits for Units	None
Aims of Unit	To develop the student's analytical techniques in the application of scientific principles to mechanical engineering situations
Special Note	The Unit is designed to be studied <i>concurrently</i> with or after TEC U75/037 Engineering Science III.

To fulfil the aims of the Unit, the book is written with reference to Engineering Science II and recognises the fact that Engineering Science III may be studied concurrently. However, it is assumed that only the statics and dynamics sections of Engineering Science II are needed as far as Mechanical Science III is concerned, although some small overlap between the two is inevitable and consequently forms some revision of essential information.

The combination of Physical Science I and Engineering Science II collectively covers forces on materials, dynamics, static equilibrium of forces and pressure in fluids in sufficient detail to be able to cover the contents of Mechanical Science III (and the relevant section of mechanics in Engineering Science III if this is studied concurrently).

The contents of Mechanical Science III are

- (1) Stress, Strain and Elasticity of Materials
- (2) Simple Theory of Bending of Symmetrical Beams
- (3) Simple Theory of Torsion of Circular Section Bars
- (4) Laws of Angular Motion

- (5) Simple Harmonic Motion
- (6) Linear and Angular Kinetic Energy
- (7) The Application of Bernoulli's Equation to Fluids in Motion.

These topics are covered essentially from first principles using these to show where any formula, which is to be applied to problems, comes from. The range of problems covered goes slightly beyond the bare minimum required, so as to allow the student an opportunity of extending his mechanical engineering knowledge for the purpose of progression in the fields of mechanical and production engineering.

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