Hydraulic Analysis of Unsteady Flow in Pipe Networks

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Preface

The reader may be interested to know how this book came to be written. The author has always found the subject of unsteady flow of great interest and throughout his career has studied it with special application. As a consequence most of his research effort has been in this area and he has guided many of his Ph.D. students into this topic also.

In 1969 an engineer from a local Consulting Engineer's office approached him requesting information concerning surge analysis methods which could take into account variations in wavespeeds caused by free bubbles in the fluid. At that moment in time the author had already developed a computer program which could analyse surge in simple rising mains but had not included this wavespeed effect. The effect was soon incorporated into the program and it was used to analyse a main which had a history of bursting to decide what was the main cause of the bursts. At the same time, unknown to the author, one of his own ex-Ph.D. students had been employed to take measurements of the pressure history of the main. When the analytic results were compared with the measured results it was found that agreement was extremely good, the only error of significance being in the timing of the pressure peaks. The actual magnitudes of the maximum and minimum pressures were excellently predicted.

Upon seeing these results the author and the engineer from the Consulting Engineer's office, Bryan Smith, decided to open an office, Hydraulic Analysis Ltd, Leeds, which would routinely undertake the analysis of proposed or existing systems. This venture turned out quite successfully and with the passage of time the firm has been called upon to analyse more and more complex systems, ranging from simple rising mains delivering sewage to a sewage works, to undersea oil pipelines such as that of the Forties field in the North Sea. The firm has been called in to analyse water supply networks for various authorities throughout the world, oil pipelines in the Middle East, most of the pipelines built or proposed for the North Sea, water injection schemes to improve oil delivery from underground strata, pipe networks in Condeeps and other complex networks such as those found in oil refineries and gas liquefaction plants. The firm has had to grow to handle this work, taking in a computer specialist, Andrew Keech,

as a partner and employing more staff.

Throughout this period it has been necessary to develop the original program and now it has reached a considerable level of sophistication.

As an academic, the author feels that the essential material in this program should be published and so he decided to write this book. It is not possible to include within the confines of one book all of the material that has gone into the program; there are many facilities which have not been included but the main material on which the program is based has been described.

The author would like to warn the reader that he has not tried to write the definitive book of waterhammer. It probably could not be written at present as the subject is still undergoing rapid development. Even so, this book is an idiosyncratic view of waterhammer and many people who have contributed greatly to the subject may feel slighted by the omission of their material or by the failure even to mention its existence. The author would like to apologise to such people and would plead, in advance, the limitations of space.

The book is idiosyncratic in other ways, techniques of finite difference integration such as those due to Lax, Wendroff and coauthors have only been given passing mention and no mention at all has been made of what the author believes is a potential technique for the future – the finite element method. However, he has demonstrated, to his own satisfaction at least, the complete adequacy of the method of characteristics and offers this as partial justification for his limited presentation of a very large, very complex subject.

Leeds, 1976

J. A. F.

Notation

Throughout this book, symbols are defined wherever they are used and these are listed below. However, variables of local interests only are not included in this list but are defined in the text.

A	area of flow
A_0	area of valve opening at time zero (chapter 1)
A_t	area of valve opening at time t (chapter 1)
a_p	area of pipe (chapter 1)
a_v	area of valve opening
A	pump constant in equation $H = AN^2 + BNQ - CQ^2$ (chapter 6)
A_e	exit area of pump impeller (chapter 6)
A_{sw}	plan area of a suction well (chapter 6)
a_e	effective valve area (chapter 9)
B	pump constant in equation $H = AN^2 + BNQ - CQ^2$ (chapter 6)
B	channel surface breadth (chapter 10)
b	mean channel breadth (chapter 10)
C_d	coefficient of discharge of a valve
c	celerity of a small pressure wave
C	constant in friction formula used in the surge tank analysis (chapter 3)
C	constant in pump equation $H = AN^2 + BNQ - CQ^2$ (chapter 6)
c_s	coefficient in the stroke equation of a servocontrolled valve (chapter 7)
C_v	specific heat of gas at constant volume (chapter 8)
C_p	specific heat of a gas at constant pressure (chapter 8)
C	electrical capacitance/unit length of a transmission line (chapter 9)
c	celerity of a small surface wave (chapter 10)
C	Chezy C (chapter 10)
C_d	coefficient of discharge of a sluice gate (chapter 10)
c_w	celerity of a surge wave in an open channel (chapter 10)

dt	time increment (infinitesimal)
dx	distance increment (infinitesimal)
dp	pressure increment (infinitesimal)
$d\rho$	density increment (infinitesimal)
dv	velocity increment (infinitesimal)
d	pipe diameter
D	pump impeller diameter
D_e	constant in the pump efficiency equation
d_{sw}	depth in a suction well
d_t	internal diameter of an air vessel
E	Young modulus of elasticity
E	pump efficiency (chapter 3)
E_e	constant in the pump efficiency equation (chapter 3)
e	internal energy of gas/unit mass (chapter 8)
E_1	constants in the characteristic forms of the unsteady gas equations as defined in text (chapter 8)
E_2	
E_3	
E	$g(j-i)$ (chapter 10)
f	Darcy f in the Darcy formula
	$hf = \frac{fLV^2}{2gm}$ (as defined in text)
f_h	hoop stress in pipe wall
f	'function of' and wave height when wave is travelling downstream (chapter 2)
F	'function of' and wave height when wave is travelling upstream (chapter 2)
F_e	constant in the pump efficiency equation
F	in chapter 8, the force acting upon the fluid/unit length of pipe
f	frequency of applied head oscillation (chapter 9)
F_n	Froude number (based on absolute velocity) (chapter 10)
g	intensity of the local gravitational field throughout the text
Gr	gradient of the pump's speed $\sim$ time rundown curve (chapter 6)
h	potential head – the sum of local pressure head and elevation of the point above an arbitrary datum.
h_f	head lost due to friction
h_s	sometimes static head, sometimes head at point S , according to context
h_n	head immediately upstream of a valve or nozzle.
h_i	potential head change caused by momentum changes (note $p_i = wh_i$)

h_{air}	pressure head of air in an air vessel expressed as height of the equivalent liquid column (chapter 3)
h_a	atmospheric pressure head
h_p	height of the base of an air vessel above the pipe centre line
h_w	as h_{air} above (chapter 7)
h_{tr}	head sensed by a pressure transducer controlling a servo-operated valve
h_{crit}	critical head at which a servo-operated valve will start to move.
$\bar{h}$	steady state head (chapter 9)
h'	unsteady head component (chapter 9)
H	amplitude of pressure head wave (chapter 9)
h_{sp}	height of the reservoir surface above the spillway crest (chapter 10)
h_w	the height of a wave crest above channel bed level (chapter 10)
I	inertia of the rotating parts of a pump and motor set (chapter 6)
i	electrical current (chapter 9)
i	$\sqrt{-1}$ (chapter 9) (see context)
i	channel bed slope – taken positively downwards (chapter 10)
j	frictional head loss/unit weight of fluid/unit length of channel (chapter 10)
K	constant in valve loss formula $h_f = K \frac{v^2}{2g}$
	$K = \frac{4fL}{d} + k$ (chapter 1 and chapter 5)
K	bulk modulus of liquid
k	constant, sometimes describing local losses,
	i.e. $h_L = k \frac{v^2}{2g}$ (due to bends, junctions etc)
	In chapter 1 also used: $k = \frac{v_0 L h_s}{g T}$
k	constant in head $\sim q$ equation $h = k q^2$, i.e. the friction formula used in the Schnyder-Bergeron method (chapter 5)
k	mean height of pipe roughnesses in the Colebrook-White formula
k_r	pump impeller head loss coefficient (chapter 6)
k_v	pump volute head loss coefficient (chapter 6)
k	spillway constant (chapter 10)
L	length, usually pipe length L
L	constant in the development of the characteristic form of the differential equations of waterhammer (chapter 7)
l_t	internal height of an air vessel (chapter 7)
L	electrical inductance/unit length of a transmission line (chapter 9)

m	hydraulic mean radius = $\frac{A}{P}$ (defined in text)
n	area ratio $\frac{a_p}{A_0}$ (chapter 1)
N	running speed of a pump in rev min^{-1} (chapter 3 and 6)
n	polytropic index (chapter 8)
p_i	pressure generated by momentum change
P	wetted perimeter
P	pump power in chapter 3 and chapter 6
p_{air}	pressure of air in an air vessel (chapter 3)
p_a	atmospheric pressure
P_{wr}	pump power in chapter 6
q	flow
q_t	flow at time t
q	heat flow/unit area (chapter 8)
$\bar{q}$	steady flow rate (chapter 9)
q'	unsteady flow component (chapter 9)
Q	amplitude of flow oscillation (chapter 9)
Re	Reynolds number = $\frac{pvd}{\mu}$ or $\frac{vd}{\nu}$
R	universal gas constant (chapter 8)
R	hydraulic resistance/unit length (chapter 9)
R_{el}	electrical resistance/unit length of a transmission line (chapter 9)
s_i	valve stroke at beginning of a Δt period (chapter 7)
s_f	valve stroke at end of a Δt period (chapter 7)
s_{req}	valve stroke required as defined by a pressure transducer (chapter 7)
s	an integer taking the value of +1 or -1 (chapters 7 and 10)
T	pipe period $\frac{2L}{c}$ (chapters 2 and 3)
T	pipe wall thickness (defined in text)
T	torque applied in the pump equation (chapter 3)
T	absolute temperature (chapter 8)
t	time
u	velocity of impeller blade tips (chapter 6)
v	mean flow velocity (chapter 1)
V_{∞}	velocity in pipe when $t \rightarrow \infty$ (chapter 1)
v_0	velocity at time zero (chapter 1)
v_t	velocity at time t (chapter 1)
V_{air}	air volume in an air vessel (chapter 3)
V_g	volume of dissolved gas

V_w	velocity of whirl at exit from a pump impeller (chapter 6)
V_r	relative velocity at exit from a pump impeller (chapter 6)
V	absolute velocity at exit from a pump impeller (chapter 6)
V_f	velocity of flow at exit from a pump impeller (Chapter 6)
V	electrical voltage (chapter 9)
V_w	velocity of a surge wave in an open channel (Chapter 10)
w	weight density of fluid
W_H	dimensionless heat parameter in the head Suter curve (chapter 6)
W_T	dimensionless torque parameter in the torque Suter curve (chapter 6)
x	distance along pipeline
X	distance along pipeline
z	elevation of the pipe centre line above datum
z	elevation of surface in a surge tank above reservoir static surface level (chapter 3)
Z_{sw}	elevation above a datum of a suction well base (Chapter 6)
z_t	the elevation above datum of the centre line of the pipeline at the point of its junction with an air vessel
Z	hydraulic impedance (chapter 9)
Z_c	characteristic impedance (chapter 9)
z	depth of the centroid of a channel cross section (chapter 10)
α	real component of γ (propagation constant) (chapter 9)
α	constant defining nature of a channel cross section (chapter 10)
β	the product $\frac{C_d A_v}{A_p} \sqrt{2g}$ (chapter 2)
β	imaginary component of γ (propagation constant) (chapter 9)
γ	pump blade angle (chapter 6)
γ	propagation constant in chapter 9
γ	ratio of specific heat of gas (chapter 8)
δ	ratio of channel cross sectional area/surface breadth (chapter 10)
Δx	distance increment (finite)
Δt	time increment (finite)
ΔV	volume increment (with subscripts to define which volume change is intended) (chapter 1)
Δp_i	pressure increment due to momentum change (finite) (chapter 1)
Δh_i	potential head change: related to Δp_i by $\Delta p_i = w \Delta h_i$ (chapter 1)
ϵ	fractional volume of free gas in liquid (chapter 5)
ϵ	ratio of a_e/a_{e_0} (chapter 9)
$\bar{\epsilon}$	steady component of ϵ (chapter 9)

ϵ'	unsteady component of ϵ (chapter 9)
ζ	the square root of the head ratio: $\left(\frac{h}{h_0}\right)^{0.5}$ (chapter 2)
ζ	the slope of the characteristic line
η	fractional valve opening: $\frac{a_v}{a_{v_0}}$ (chapter 2)
θ	angle in Suter presentation of four quadrant pump characteristic $\theta = \tan^{-1} \left(\frac{N}{N_s} \frac{Q_s}{Q} \right)$ (chapter 6)
λ	constant in the characteristic formulation of the waterhammer equations (chapter 4)
λ	$\alpha \frac{d}{B} \frac{\partial B}{\partial x}$ (chapter 10)
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid = $\frac{\mu}{\rho}$
π	3.14159
ρ	mass density
ρ	Allievi characteristic: $\frac{cv_0}{2gh_0}$ (chapter 2)
τ	surface tension coefficient in chapter 5
τ	viscous shear stress in chapter 2 and chapter 10
χ	the compound line produced by the summing of two eages in the Schnyder – Bergeron method (chapter 2)
ψ	the valve characteristic in the Schnyder – Bergeron method (chapter 2)
ψ	phase angle (chapter 9)
Ω	angular velocity of pump impeller (chapter 3 and chapter 6)
Ω	angular velocity of the applied head oscillation (chapter 9)