Managing Mathematically

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and

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Editor's Introduction

Alan Hale

This series is a direct outcome of the opportunities and challenges created by the rapid expansion of higher and further education in the past decade.

The expansion involved changes in the structure of advanced education, through the CNAA, the Polytechnics, the Regional Management Centres and the professional bodies which encouraged staff to develop new and experimental teaching. Substantial changes have taken place in the definition, scope and methodologies of the social, administrative and management sciences leading to modifications in the presentation of these subjects. Many new full-time students and staff have questioned traditional approaches and methods and have established more open discussion and debate on their courses. Demands for qualified manpower led to an expansion in part-time education and increased questioning by students in full-time jobs of the relevance of their studies.

Each of these developments has had a profound impact on the structure and content of courses and given fresh impetus to the discussion and modification of curricula and teaching methods in polytechnics, universities and colleges of further education. The editor and authors of the books in this series have made a deliberate attempt to respond to these changes.

The books set out to provide a comprehensive and up-to-date introduction to the ideas and methods of their subjects for specialist and non-specialist students in fields such as business and management studies, social science and administration. Their aim is to help students who have little or no previous knowledge of them to achieve a mastery of the scope and basic techniques of their subjects and to use them critically and with imagination for further study or for practical professional applications. They also seek to make some contribution to discussions of teaching and learning problems in their field.

Many introductory books present their subjects as a coherent body of knowledge of which the logic is self-evident and the concepts and methods clear to the careful reader. Students do not always find this so. Confronted as they are by a well-established discipline which has

developed a particular method that may not bear any obvious relationship to the way in which they have been accustomed to think or to realities as they see them, students often have difficulty in comprehending the significance and detail of the forms of analysis it employs.

The editor and authors of the series felt that they should not take for granted the 'self-evident logic' of their subjects, but try to demonstrate to readers the ways in which their disciplines provide an effective framework for the analysis of problems in their field. When abstractions or concepts are introduced their functions and limitations are explained. Where methods or techniques are described the authors show why they take the form they do and the ways in which they may be used for particular tasks of analysis.

Students often criticise courses because their subjects or parts of them do not have any obvious, or immediate, practical applications. They may present what appear to be unnecessarily complicated ways of dealing with quite straightforward problems or, paradoxically, they may be regarded as over-simplifying or ignoring difficulties which are experienced in real situations. Criticisms of this sort are due to misunderstandings about the nature of subjects. Systems of knowledge provide generalisations which are derived from a variety of abstractions and models. Some of these yield tools of description and analysis that have direct applications. Others suggest ways of looking at problems that, however fruitful, may have only limited or indirect applications.

The authors have tried to make clear the relevance of their subjects. Where concepts and methods have direct applications they show how, and under what circumstances, they can usefully be applied. Where they are of indirect use they show how a process of simplification may isolate and draw attention to the important characteristics of a complex problem, or how the study of complicated or abstract aspects of a problem may throw fresh light on it.

The authors have rejected a view, reflected in many basic textbooks, that students at an introductory level should concentrate on 'learning up' the information and techniques of their subjects and not be troubled with discussions of concepts and analytical method. This 'descriptive' approach, which divorces the study of techniques from that of conceptual and analytical structures, makes it more rather than less difficult for students to appreciate how a technique has developed, why it takes a particular form and how it functions. As a result students can spend a considerable time trying to understand, with limited success, a method of describing and presenting information or a method of analysis and fail to achieve any real facility in using it. The discussion of concepts and analytical method also acquaints a student with some of the difficulties and controversies surrounding the ideas

and techniques he is studying. Without such knowledge he is unlikely to appreciate their limitations or establish any real ability to discriminate between alternative approaches and methods.

One of the more important aims of education is to develop a student's capacity to formulate and solve theoretical and practical problems. It is clear that few business and administrative problems are in practice separable into the neat categories represented by disciplines such as economics, accounting, law, sociology, psychology and computing. But most courses are based on combinations of studies in these and similar discrete disciplines which are rarely effectively integrated. It is recognised that the development of bodies of knowledge which provide rigorous rather than superficial integrative approaches will be a long and difficult task. The editor and authors of the books in this series are aware of this problem and within their limitations have attempted to indicate points at which contributions from other disciplines are necessary to the analysis of the problems with which they are dealing. It is thus hoped that in the long run the series will make some contribution to the development of interdisciplinary approaches.

The problems outlined above are common in the teaching and learning of many subjects which, although emerging historically as systems for analysing and solving practical problems, have developed advanced methodologies and a logical order of presentation that may not bear an obvious relation to the practical problems with which they are supposed to be concerned.

Although mathematics has long been used in the analysis and solution of scientific and technical problems and is now applied in a wide variety of management activities many still regard it as incomprehensible or irrelevant. Previous experience of mathematics has led some to believe that it is difficult and dull subject in which complicated techniques are used to solve problems that are either too trivial or too advanced to interest them. Others see little need for it in their work where well-tried non-quantitative methods appear to be quite adequate.

It is surprising how often those who avoid mathematics do in their daily experience estimate variables, assess risks and play games which involve the sort of mental activities with which mathematics is concerned. Perhaps difficulties arise because mathematics uses unfamiliar symbolic language to deal with familiar situations and that it employs rational and logical approaches to problems that are often dealt with in a fairly intuitive way.

The power of mathematics is that it enables significant characteristics of problems to be identified more precisely and represented more simply than can be done by non-quantitative means. Problems in management and administration are increasingly dealt with by the use

of symbolic and quantitative methods and learning difficulties in subjects such as economics or accounting often arise because of a lack of basic mathematical education. The authors of this book have tried to present mathematics in an interesting and relevant way and it is hoped that it will help many who are uncertain about the subject to develop an ability to use basic mathematics with ease and confidence.

The editor and authors of the books in this series are conscious of their limitations in attempting to implement their ideas in writing and teaching and do not suppose that their presentation will solve students' learning problems. They do not ignore the critical importance of motivation and sustained and disciplined study as factors in effective learning. But they felt that if subjects were presented in a way that made their form and justification explicit rather than implicit this would aid teaching and learning.

In seeking to achieve their aims the books in the series have been subjected to a great deal of critical scrutiny. Each is written by more than one author. This has enabled authors to combine a comparison of views with a considerable, and sometimes uncomfortable, degree of mutual criticism. The editor and authors have all, in recent years, had considerable experience of designing and teaching new CNAA honours degree, diploma or professional courses. Their manuscripts have been discussed with colleagues in education and the professions and have been tested in classes with students.

My thanks as editor are owed to the authors who responded to my request to write the books in the form that I have outlined. This has involved them in an extremely demanding process. The fact that we shared some basic assumptions about education and learning was of great help. The editors and staff of Macmillan with whom we have worked have shown great patience and could not have done more in difficult circumstances to encourage the series. My thanks are also due to George Brosan, Maurice Peston and Bruce May, who through many discussions have significantly influenced my educational ideas, and to my wife Diane, who has kept a discerning eye on my activities and the progress of the series.

February 1976

ALAN HALE

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Preface to the Student

Why You Need This Book

You need this book to help you to BE MATHEMATICAL. Surely it is no longer respectable for a businessman to say in a jovial and rather complacent fashion ‘of course I know nothing about all this mathematics business ...’. The increasing use of mathematics techniques and arguments in business means that at the very least every businessman should know enough to appreciate the contribution mathematics can make, and the effect it has, on his management role. But more than that, he needs to be numerate enough to participate in this mathematical activity in order to derive the full benefits from it.

This book has been written to help you develop mathematically and so reach a higher level of numeracy. We will not ask you to perform mathematical tricks even more elaborate than those you have learnt already at school. Our object is not to make you into a MathematicIAN; we want you to be mathematiCAL. Being mathematical is an attitude of mind, nothing to do with being especially adept at adding and subtracting, and enables you to be analytical in your judgement of problem situations.

For example: a queue in a supermarket would be seen in terms of colours, shape and their interaction. From the mathematical point of view the same queue would be seen quantitatively in terms of the number of people in the queue, the rate at which they are served and how the two are related.

How to Read This Book

Unlike many mathematics books (see Chapter 9) this one is to READ, starting from page 1. To read this book you need:

a pen
a lecturer
a library

Mathematics is an active pursuit, like swimming. You would not

expect to be able to swim after reading a book; you need to try out the instructions and through these to learn new skills. So here and there in the text we ask you to pick up your P E N and D O mathematics.

Your mathematical progress is marked by the questions of increasing complexity you ask yourself. We shall ask you questions directly and suggest others for you to ask your lecturer. The answers will not always be found within the text, for the book was not intended to replace a lecture course, but to be used in conjunction with it. Inevitably you will have difficulties and you need a LECTURER to help you with them.

All the material in the text is designed to help you develop mathematically. It is a selection of the mathematics and statistics which is used extensively in business and should equip you to study further from other books. You need a LIBRARY to investigate in greater depth topics that have been introduced in this book, and to find out about ideas and techniques we have not included.

Your Examinations and You

Your mathematical progress in college will be measured by the success you have in examinations. This is a problem that you cannot ignore. Chapter 10, Examination of Examinations, demonstrates how examination questions should be interpreted enabling you to recognise what the examiner expects to be included in the solutions.

Synopses of Chapters

Preface To the Student

Preface To the Lecturer

1 WHAT'S YOUR PROBLEM?

Using real situations, the progression is made from solving the well-determined undiscipline problems at school to solving the complex multidiscipline problems of the business world. From this is demonstrated the needs of the businessman if he is to operate efficiently and effectively, and hence the *raison d'être* of the book.

2 DATA: WHAT CAN IT SAY?

A problem situation is used to demonstrate the need for data collection. The methods of interpreting data are grouped into five levels, including visual, statistical measures and probability. The contribution each successive level can make to the understanding of behaviour in the problem situation is assessed.

3 LINE UP THE FUNCTION

The straight-line model arises naturally from the business environment and is investigated both from the mathematical and statistical point of view. The idea and notation of 'function' is developed from the study of the line and its business applications. These ideas are used to introduce the idea of slope and its invariance under change of scale. A further extension gives a business meaning to the solution of equations.

4 FUNCTION EXTENDED

Slopes of curves are studied in detail and the notation of differential calculus is introduced.

Calculus ideas are developed to solve maxima and minima problems, particularly in the case of finding the best-fitting straight line to a set of data. Finally Newton's method of solving equations using an iterative method introduces the student to algorithms, flow diagrams and rates of convergence.

5 MODELLING SYSTEMS

The behaviour described by an equation and inequality is discussed. The same system is represented by models of varying complexity by making different assumptions. The difficulties of obtaining the data for each model are considered. The solution to a linear programming model is found and its validity investigated.

6 MODEL ANALYSIS

The choice of a model is discussed by introducing the concept of total error. The simulation approach to problem-solving is described, illustrated by a simplified but real problem, using one of the models of Chapter 5.

7 COPING WITH UNCERTAINTY

Uncertainty is discussed in terms of probability and the interpretation of collected data. A quality-control problem is used to develop the statistical ideas of risk and significant difference. Stress is placed on the generality of the ideas of statistic, parameter and distribution, rather than on the study in detail of any one particular probability model.

8 BE SURE OF THE DATA

Various agencies collect data and present the results in condensed form to decision-makers. This chapter discusses the problems of interpreting such results, planning successful data collection projects and using mathematical specialists and their techniques effectively.

9 AFTERMATHS

The reader is invited to examine his mathematical attainment and to consider ways of continuing development. Attention is drawn to topics not covered in the text and advice is given on how to deal with them. Some useful literature is listed.

10 EXAMINATION OF EXAMINATIONS

Most examination questions in quantitative methods are artificial exercises in mathematical and statistical techniques and as such should be relatively easy to identify and solve.

Examination questions which have been set on business diploma and degree courses are discussed and solved in detail. The solutions are designed to bring out the underlying mathematical principles, explore the relevance of the assumptions and give the effective methods of solution under examination conditions.

To the Lecturer

The Style of the Book

This book looks quite different from most other basic texts in mathematics and statistics, which have distinct chapters on surveys, significance testing, sampling, the Normal distribution, the straight line etc. . . . These areas are included in this book, but not separately. The approach used is to consider realistic problems and to introduce the appropriate mathematical technique as the need arises. We believe this helps the student to understand the advantages of looking at problems mathematically, and why in a particular situation one technique is used in preference to another.

The Book and the Student

The book is complementary to, not a replacement for, the lectures and students will benefit from reading it throughout the course. It is designed to develop a mathematical awareness in the student, encouraging him to become involved in his learning situation and to approach your lecture material in a questioning frame of mind.