VIBRATIONS OF CAM MECHANISMS

Philips Technical Library

VIBRATIONS OF CAM MECHANISMS

Consequences of their design

M. P. KOSTER

Macmillan Education

ISBN 978-1-349-02457-5 ISBN 978-1-349-02455-1 (eBook)
DOI 10.1007/978-1-349-02455-1

English edition © N. V. Philips' Gloeilampenfabrieken, Eindhoven, 1974
Softcover reprint of the hardcover 1st edition 1974 978-0-333-17732-7

All rights reserved. No part of this publication may be reproduced or transmitted
in any form, or by any means, without permission.

SBN 333 17732 0

First published 1974 by
THE MACMILLAN PRESS LTD
London and Basingstoke
Associated companies in New York Melbourne
Dublin Johannesburg and Madras

PHILIPS

Trademarks of N. V. Philips' Gloeilampenfabrieken

PREFACE

The book is intended both for those who are interested in the theoretical backgrounds of machine dynamics and for the engineering designer.

The book, initiated by the author's design work relating to production machines, contains a theoretical analysis and rules of design which are applicable in the field of the design of high-accuracy, fast-running machines, without recourse to computer-aided programs.

These rules are formulated in order to design machines which fulfil requirements concerning positional accuracy and dynamic load; they can be used by the designer without specialised knowledge of the theory of dynamics.

The dynamic behaviour of machines can be predicted if the mechanism is represented by a mathematical model. In this book, cam mechanisms and their driving components, i.e. the camshaft, the reduction gear, and the driving asynchronous motor are investigated, but the methods and results are of wider applicability.

Transient vibrations characterise the dynamic behaviour of cam mechanisms. In present-day machines the main cause of follower vibration is often inadequate shaft rigidity. Several mathematical models have been developed and tested. Detailed analysis of machine vibrations can be obtained by means of a digital simulation program based on a model with four degrees of freedom. Follower and shaft vibrations and the effects of nonlinear phenomena, i.e. backlash, squeeze, and impact are simulated. Based on a model with one degree of freedom rules of design concerning the cam follower linkage and the driving components are drawn up. A number of examples are given.

M. P. Koster

Eindhoven,
May 1974

ACKNOWLEDGMENT

This book first appeared in 1973 as a thesis. Sincere thanks are due to all who have contributed to these investigations, especially to prof. dr ir J. D. Janssen and prof. ir W. van der Hoek for their support during the preparation of the thesis, to Mr. J. A. M. Spapens for his enthusiastic cooperation, to Mr. J. A. Versantvoort for his advice relating to the application of the software package COSILA, and to Mr. H. G. E. Wallace for his critical remarks and conscientious revision of the text.

M. P. Koster

**Eindhoven,
May 1974**

CONTENTS

SYMBOLS

1. INTRODUCTION	1
1.1. Mechanisms	1
1.2. Why the interest in the dynamics of cam mechanisms	2
1.3. State of the art	3
1.4. The aim of the present investigation	9
1.5. Survey of the present investigation	10
2. MODELLING	13
2.1. Introduction	13
2.2. Degrees of freedom	13
2.2.1. Continuous systems	13
2.2.2. Discrete systems	15
2.2.2.1. Static and dynamic coupling	15
2.2.2.2. Static coupling only	25
2.3. Dynamic model	29
2.3.1. Transmission ratio	29
2.3.2. Addition of motions	32
2.3.3. Dynamic models of machine components	33
2.3.4. Example	33
2.4. Reduction	35
2.4.1. Reduction of linear dimensions	35
2.4.2. Reduction of mass	35
2.4.3. Reduction of stiffness	36
2.4.4. Reduction of a linear, viscous damper	37
2.4.5. Reduction of viscosity	37
2.4.6. Example	38
2.5. Details of the dynamic model	38
2.5.1. Variable transmission ratio $\tan \alpha$	40
2.5.2. Variable magnitudes of m_{β}' and c_{β}'	40
3. SINGLE-DEGREE-OF-FREEDOM MODEL WITH CONSTANT FICTITIOUS ANGULAR VELOCITY	42
3.1. Introduction	42
3.2. Assumptions	42
3.2.1. Constant fictitious angular velocity	42
3.2.1.1. Infinitely rigid camshaft	42
3.2.1.2. Backlash-free transmission	43
3.2.1.3. Constant input angular velocity	43

3.2.2. Lumped mass	43
3.2.3. Decay of residual vibration	43
3.2.4. No damping during cam rise	44
3.3. Dynamic response	44
3.4. Damping	47
3.5. Maximum and zero points	48
3.6. Backlash	48
3.7. Many degrees of freedom	50
3.7.1. Transfer function	50
3.7.2. Natural frequencies	52
3.7.3. Response	52
3.8. Conclusions	56

4. SINGLE-DEGREE-OF-FREEDOM MODEL WITH VARIABLE FICTITIOUS ANGULAR VELOCITY, "CAMSHAFT-1"

4.1. Introduction	58
4.2. Assumptions	59
4.2.1. Constant input angular velocity	59
4.2.2. Constant mass	59
4.2.3. Variable stiffness	60
4.2.4. No damping during cam rise	60
4.2.5. Constant pitch radius	61
4.3. Description of the model	62
4.3.1. Equations	62
4.3.2. Dimensionless numbers	64
4.3.2.1. Introduction	64
4.3.2.2. Definitions	64
4.3.2.3. Dimensionless parameters	65
4.3.2.4. Dimensionless equations	65
4.4. Methods of solution	66
4.4.1. Approximative method	66
4.4.2. Numerical method	68
4.5. Results of the CAMSHAFT-1 program	70
4.5.1. Acceleration phase	70
4.5.2. Deceleration phase	71
4.5.3. Natural frequency	71
4.5.4. Residual vibration	73
4.5.5. Response graphs	74
4.6. Simple method for calculating the lowest natural frequency	74
4.7. Conclusions	77

5. TWO-DEGREES-OF-FREEDOM MODEL, "CAMSHAFT-2" . .	79
5.1. Introduction	79
5.2. Assumptions	79
5.2.1. Two degrees of freedom	79
5.2.2. Constant input angular velocity	81
5.3. Description of the model	81
5.3.1. Equations	81
5.3.2. Dimensionless numbers	82
5.3.2.1. Definitions	82
5.3.2.2. Dimensionless parameters	83
5.3.2.3. Dimensionless equations	83
5.4. Method of solution	84
5.5. Results of the CAMSHAFT-2 program	88
5.5.1. Shaft flexibility	88
5.5.2. Acceleration phase of the cam rise	88
5.5.3. Deceleration phase of the cam rise	88
5.5.4. Residual vibration	88
5.5.5. Dynamic behaviour of the shaft	88
5.5.6. Response graphs	90
5.5.7. Natural frequencies	90
5.5.7.1. Highest natural frequency	93
5.5.7.2. Lowest natural frequency	94
5.6. Extension to more degrees of freedom	94
5.7. Conclusions	98
 6. FOUR-DEGREES-OF-FREEDOM MODEL, "DYNACAM" . .	 100
6.1. Introduction	100
6.2. Assumptions	100
6.2.1. Four degrees of freedom	100
6.2.2. Damping	100
6.2.3. Backlash and squeeze	100
6.2.4. Follower jumping	100
6.2.5. Nonconstant angular velocity of the driving machine . .	101
6.3. Numerical calculations	102
6.4. The purpose of the DYNACAM model	102
6.5. Equations of the cam mechanism	102
6.5.1. Input motion	102
6.5.2. Backlash in reduction gear	102
6.5.3. Torsion of the camshaft	105
6.5.4. Deflection of the camshaft parallel to the direction of motion of the follower roller	106

6.5.5.	Deflection of the camshaft perpendicular to the direction of motion of the follower roller	106
6.5.6.	Frictional forces acting on the rotating system	106
6.5.7.	Rotation of the cam	107
6.5.8.	Displacement of the cam parallel to the direction of motion of the follower roller	107
6.5.9.	Displacement of the cam perpendicular to the direction of motion of the follower roller	107
6.5.10.	Cam curve	107
6.5.11.	Instantaneous slope of the cam curve, instantaneous pitch radius, instantaneous roller displacement, and instantaneous roller velocity	107
6.5.12.	Backlash of the roller in its groove	109
6.5.13.	Deformation of the follower linkage	111
6.5.14.	External friction of the follower linkage	112
6.5.15.	Motion of the follower output member	113
6.5.16.	Follower spring	113
6.5.17.	External forces	113
6.5.18.	Damping	113
6.5.19.	Integrals	113
6.5.20.	Initial conditions	114
6.6.	Block diagram	117
6.7.	Equations of the asynchronous motor and the reduction gear	117
7.	ANALYSIS OF MACHINE DYNAMICS	120
7.1.	Introduction	120
7.2.	Particulars of the test mechanism	120
7.3.	Measuring equipment	124
7.4.	Follower-response measurements explained by means of simulation	125
7.4.1.	Static characteristics	125
7.4.1.1.	Backlash	125
7.4.1.2.	Stiffness	125
7.4.1.3.	Hysteresis	128
7.4.2.	Measured dynamic characteristics	131
7.4.3.	Simulation with the DYNACAM program	131
7.4.4.	Traversing of backlash	132
7.4.5.	Damping	143
7.4.6.	Squeeze	147
7.4.7.	Conclusions	147
7.5.	Follower jumping	147
7.6.	Flexible shaft	147

8. COMPARISON OF THE RESPONSES OF THE VARIOUS DYNAMIC MODELS WITH THOSE OF REAL MECHANISMS, AND RULES OF DESIGN	151
8.1. Introduction	151
8.2. Comparison of the responses of the various dynamic models of cam mechanisms with those of real mechanisms	151
8.2.1. Four-degrees-of-freedom model	151
8.2.2. Damping	156
8.2.3. Follower spring	156
8.2.4. Two-degrees-of-freedom model	156
8.2.5. Single-degree-of-freedom model with variable fictitious angular velocity	157
8.2.6. The envelope curve $U_0(\tau, F_a)$	157
8.2.7. Constant pitch radius	157
8.2.8. Conclusions	161
8.3. Selection of the asynchronous motor	162
8.4. Alternative cam curves	163
8.5. Rules of design	168
8.6. Conclusions	171

9. CONCLUSIONS AND RECOMMENDED FURTHER INVESTIGATIONS	172
9.1. Conclusions	172
9.2. Recommended further investigations	172
9.2.1. Roller vibration	172
9.2.2. Basic cam curve	173
9.2.3. Damping	173
9.2.4. Dynamic behaviour of the frame	174
9.2.5. Considerations of practical design	174
9.2.6. Dynamic behaviour of multibar linkages	174

APPENDIX 1. Dynamic models of machine components	175
--	-----

APPENDIX 2. Nominal characteristics of the cycloidal and the modified sine cam curve	179
A.2.1. Introduction	179
A.2.2. General	179
A.2.3. Cycloidal cam curve	180
A.2.4. Modified sine cam curve	182

APPENDIX 3. Rayleigh's method for the calculation of the lowest	
natural frequency	184
A.3.1. Introduction	184
A.3.2. Rayleigh's method	184
A.3.3. Exact method for lumped masses	184
A.3.4. Approximative single-degree-of-freedom system	185
A.3.5. The accuracy of Rayleigh's method in the two-degrees-of-free-	
dom system	185
A.3.6. Measure of the accuracy of Rayleigh's method	186
A.3.6.1. Multi-degree-of-freedom systems	186
A.3.6.2. Intermediate masses exceed the output member mass	188
A.3.6.3. Directive for calculation of the lowest natural fre-	
quency	191
 APPENDIX 4. Squeeze	 192
A.4.1. Two-dimensional, steady-state viscous flow	192
A.4.2. Squeeze	193
 APPENDIX 5. Asynchronous motor	 195
A.5.1. Introduction	195
A.5.2. Electromagnetic torque	195
A.5.3. Equations of the motor	197
A.5.4. Methods of solution	198
A.5.4.1. Motor considered as a second-order system	198
A.5.4.1.1. Numerical solution	198
A.5.4.1.2. Analytical solution	198
A.5.4.2. Motor considered as a first-order system	203
A.5.4.3. Comparison of the actual response characteristics of	
the asynchronous motor with those of the various	
dynamic models	205
A.5.4.3.1. Second-order dynamic model	205
A.5.4.3.2. Efficiency of the reduction gear	207
A.5.4.3.3. Spring load	207
A.5.4.3.4. Damping	207
A.5.4.3.5. Analytical calculation, second-order system	208
A.5.4.3.6. Analytical calculation, first-order system .	209
A.5.4.3.7. Motor-speed variations affecting the cam	
follower response	209
A.5.4.3.8. Conclusions	210
 APPENDIX 6. Program "DYNACAM"	 211

APPENDIX 7. Calculations of dynamic models	215
A.7.1. Introduction	215
A.7.2. Indexing mechanism	215
A.7.3. Cam follower mechanism	218
A.7.3.1. Four-degrees-of-freedom model	218
A.7.3.2. Two-degrees-of-freedom model	221
REFERENCES	223

SYMBOLS

Indices of symbols occurring frequently are listed separately at the end of this nomenclature. Numbers between parentheses refer to equations in which the symbols have been used or defined.

α	angle of pressure exerted by the cam (2.52)	(rad)
α_{mc}	(= $\arctan(h_m/R_{1/2}\beta_m)$) mean angle of pressure exerted by the cam (4.13)	(rad)
α_t	angle of pressure exerted by gear teeth (6.71)	(rad)
β	angular position of the cam	(rad)
β_m	angle of the cam curve	(rad)
γ	density	(kg/m ³)
δ	difference: (1) increment (4.28); (2) deviation	
δ	slip of the asynchronous motor (8.1)	
$\delta \dots$	increment (4.23)	
δ_k	critical slip (A.5.3)	
δ_{lim}	maximum allowable slip (8.1)	
ε	contact ratio (6.3)	
η_g	efficiency of reduction gear (A.5.34)	
η	dynamic viscosity of lubricant (A.4.2)	(Ns/m ²)
θ	(1) (= $\Delta T/2$) half the time step ΔT (4.24), (5.21); (2) instant of occurrence of maximum nominal acceleration of the modified sine cam curve (A.2.17)	
λ	eigenvalue (2.50)	
μ	angle of friction (6.71)	(rad)
ν	coefficient of friction (6.13)	
ξ	constant (4.22)	
ρ	radius of curvature of the cam (6.31)	(m)
ρ_{pitch}	radius of curvature of the pitch curve (6.30)	(m)
σ_p	pressure in fluid (A.4.1)	(N/m ²)
σ_s	shear stress in fluid (A.4.2)	(N/m ²)
τ	T_c/t_m (3.3)	
τ_{cl}	T_{cl}/t_m (A.5.23)	
τ_m	T_m/t_m (A.5.24)	
ϕ_v	volume flow (A.4.4)	(m ³ /s)
ϕ	(= φ/β_m) dimensionless angular position of the driving machine	
φ	angular position of the driving machine, reduced at camshaft	(rad)

φ_m	angular position of the driving machine	(rad)
φ_t	angular position of the shaft at the side facing the reduction gear	(rad)
χ	output angle	(rad)
ψ	angle of rotation (2.1.b)	(rad)
Ω_{fic}	(= $\omega_{fic} t_m/\beta_m$) dimensionless fictitious angular cam velocity	
ω	(= $\dot{\varphi}$) angular velocity of driving machine, reduced at camshaft (A.5.9)	(rad/s)
$\Delta\omega$	= $\omega - \omega_s$ (A.5.6)	(rad/s)
ω_e	natural (radial) frequency	(rad/s)
ω_{eh}	highest natural frequency (5.46)	(rad/s)
ω_{el}	lowest natural frequency (5.46)	(rad/s)
ω_{ehu}	highest natural frequency in uncoupled situation (5.45)	(rad/s)
ω_{elu}	lowest natural frequency in uncoupled situation	(rad/s)
ω_m	(= $\dot{\varphi}_m$) angular velocity of motor shaft	(rad/s)
ω_{mains}	mains (radial) frequency (A.5.3)	(rad/s)
ω_{fic}	fictitious angular cam velocity (1.3)	(rad/s)
ω_s	constant angular velocity of camshaft (A.5.9)	(rad/s)
A	cross-sectional area	(m ²)
AA	= $(\varphi_t - \varphi)^{-3/2}$ (6.7)	(rad ^{-3/2})
a_{ij}	coefficient (2.28), (2.29)	
a_i	component of displacement vector (4.36)	
B	(= β/β_m) dimensionless position of cam	
BB	= $(\varphi + s_t - \varphi_t)^{-3/2}$ (6.8)	(rad)
ΔB	(1) increment of B (5.34); (2) (= $(\varphi_t - \beta)/\beta_m$) dimensionless shaft wind-up	
b	length of cylindrical surfaces subject to squeeze (A.4.11)	(m)
CC	= $(h_r' - h_c')^{-3/2}$ (6.39)	(m ^{-3/2})
$\bar{C}$	stiffness matrix (2.48)	(N/m)
c	stiffness	(N/m)
c_c	constant stiffness (4.3)	(N/m)
c_{tan}	tangential stiffness (4.4)	(N/m)
D	d/dt, differential operator	(s ⁻¹)
DD	= $(h_c' + s_r' - h_r')^{-3/2}$ (6.40)	(m ^{-3/2})
d	coefficient of damping	
E	Young's modulus	(N/m ²)
$\bar{E}$	unit matrix (2.49)	
F	force	(N)
F_a	shaft-rigidity factor (4.14)	

F_b	impact force	(N)
F_{ex}	external force	(N)
F_v	flywheel factor (5.14)	
F_o	follower-spring preload	(N)
f	displacement of foundation	(m)
G	(1) modulus of shear; (2) linearised static characteristic (A.5.1)	(N/m ²) (Nm s/rad)
G_o	slope of tangent to static characteristic at zero torque (A.5.4.a)	(Nm s/rad)
H	(= h'/h_m') dimensionless cam-curve displacement	
h	cam-curve displacement (A.3.2)	(m)
$\bar{h}$	input displacement vector (2.48)	(m)
h_c	displacement of the centre of a backlash-free follower roller (2.52)	(m)
h_m	stroke at cam	(m)
h_r	position of the centre of the roller	(m)
I	moment of inertia of a cross-section	(m ⁴)
I_p	polar moment of inertia of a cross-section	(m ⁴)
i	transmission ratio	
i_t	transmission ratio of reduction gear (A.5.6)	
i_x	transmission ratio of the follower linkage	
J	polar mass moment of inertia of the cam	(kg m ²)
J_m	polar mass moment of inertia of the driving machine (A.5.7)	(kg m ²)
$\bar{K}$	stiffness matrix (2.48)	(N/m)
k	(= $G I_p / l$) torsional stiffness	(Nm/rad)
l	length of bar, beam, or shaft	(m)
M	torque in camshaft (A.2.5)	(Nm)
$\bar{M}$	mass matrix (2.48)	(kg)
M_e	electromagnetic torque of driving machine (A.5.1)	(Nm)
M_k	critical torque of the electric motor (A.5.4.a)	(Nm)
M_m	external torque on the driving machine (A.5.7)	(Nm)
m	mass	(kg)
m_t	modulus of output gear wheel (6.15)	(m)
m_{tan}	mass in tangential direction (5.51)	(kg)
N	power (8.3)	(W)
n	integer	
P	(= M_{ena}/M_{max}) ratio of overdimensioning (A.5.15)	
P_β	(= $M/(\beta_m c_\beta' R_b'^2)$) dimensionless torque	
p	number of pairs of poles (A.5.4.a)	
Q	force in a nodal point (2.28), (2.29)	(N)
Q^*	generalised force (2.27)	(N)

q	relative damping (1.2)	
$R(\beta)$	instantaneous pitch radius of cam curve (2.53)	(m)
R_B	(= $R'(\beta)/h_m'$) dimensionless instantaneous pitch radius	
R_e	effective radius of curvature (A.4.8)	(m)
R_b	base pitch radius (4.1)	(m)
$R_{1/2}$	(= $R_b + \frac{1}{2} h_m$), mean pitch radius (4.1)	(m)
R_r	radius of roller (6.31)	(m)
r_c	(= $v_b t_m/h_m'$) dimensionless impact velocity (3.16)	
S_r	(= s_r'/h_m') dimensionless backlash of the roller in the cam groove	
S_t	(= s_t/β_m) dimensionless backlash in reduction gear	
S_f	instantaneous minimum film thickness (A.4.10)	(m)
s	backlash (3.16)	(m)
s_f	instantaneous film thickness (A.4.3)	(m)
$s_{\theta 0}$	initial position of shaft (6.61)	(rad)
s_{x0}	initial position of roller (6.56.b)	(m)
T	(1) (= t/t_m) dimensionless time (A.2.1); (2) kinetic energy (2.7)	(Nm)
T_e	period of natural vibration (3.3)	(s)
T_{e1}	electrical time constant (A.5.3)	(s)
T_m	mechanical time constant (A.5.11)	(s)
t	time	(s)
t_c	time of a shaft cycle	(s)
t_d	dwelling time	(s)
t_m	cam rise time (3.2)	(s)
U_{sn}	(= U_{sn}/h_m') dimensionless impact amplitude (3.11)	
$\dot{U}_{sn}$	(= $\ddot{u}_{sn} t_m^2/h_m'$) dimensionless amplitude of the acceleration at impact (3.12)	
U_0	(= U_0/h_m') dimensionless positional error (3.5)	
$\dot{U}_0$	(= $\ddot{u}_0 t_m^2/h_m'$) dimensionless amplitude of the residual acceleration (3.6)	
u	(= $x - h_m'$) instantaneous positional error (3.4)	(m)
u_{sn}	impact amplitude (3.11)	(m)
$\ddot{u}_{sn}$	amplitude of the acceleration at impact (3.12)	(m/s ²)
u_0	amplitude of the positional error (3.5)	(m)
$\ddot{u}_0$	amplitude of the residual acceleration (3.6)	(m/s ²)
V	potential energy (2.5)	(Nm)
v	velocity in fluid film parallel to squeezing surfaces (A.4.3)	(m/s)

v_b	impact speed (3.13)	(m/s)
VR	squeeze term related to squeeze between the roller and the groove wall (6.38)	(Ns m ^{3/2})
VT	squeeze term related to squeeze in the reduction gear (6.6)	(Ns m ^{3/2})
w	squeeze velocity (A.4.5)	(m/s)
X	(= x/h_m) dimensionless position of the output member	
x	position of output member (1.2)	(m)
x_d	dynamic displacement function	(m)
x_e	elastic displacement function (2.2)	(m)
x_s	displacement function with dynamic coupling neglected	(m)
$\mathbf{x}$	displacement vector (2.48)	(m)
z_t	number of teeth of the output gear wheel (6.15)	

Indices

β	refers to rotation of the cam
a	refers to magnitudes reduced at the camshaft
e	refers to: (1) elastic displacement; (2) electromagnetic torque (A.5.1)
eq	refers to equivalent
f	refers to foundation
l	refers to longitudinal motion (2.1.a)
max	refers to maximum
me	refers to mean
min	refers to minimum
n	refers to rated (8.3)
r	refers to roller
ri	refers to rigid
s	refers to follower spring (6.45)
t	refers to gear-wheel teeth
tr	refers to transverse motion (2.1.c)
w	refers to friction (6.13)
x	refers to displacement of the follower output member
y	refers to tangential deflection
z	refers to deflection in the direction of motion of the follower roller
0	refers to initial condition
'	refers to reduced magnitude
$\wedge$	refers to amplitude
.	refers to d/dt (s ⁻¹) or d/dT, for example $\dot{h} = dh/dt$; $\dot{H} = dH/dT$