ESSENTIAL SOLID MECHANICS

Theory, worked examples
and problems

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and Mechanical Engineering
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To my father

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PREFACE

The analysis of force, stress and deformation in engineering components is traditionally covered in the study of strength of materials and theory of structures. However, there is so much common ground between these two disciplines, particularly in the early stages, that it seems much more appropriate to treat them as the single subject referred to here as Solid Mechanics.

The purpose of this book is to establish, in concise form, the bases of solid mechanics required by mechanical, civil and structural engineering undergraduates in the first half of a university or polytechnic degree course. The format consists of the elements of the theory for a particular topic followed by a number of worked examples illustrating the application of the theory. Each chapter ends with a selection of problems (with answers) which the student can use for practice. The examples and problems are typical of those set to first and second year undergraduates and have been collected and adapted from a large number of sources during many years of teaching in the field. The precise origin of the questions is unknown but a general acknowledgement is given here. The author alone is responsible for the solutions and answers. There are a total of over two hundred problems in the book of which about half have fully worked solutions.

This book presents all the necessary theory in a compact form and for reasons of space the broader background to the subject has obviously had to be omitted. The student is well advised to extend his knowledge by further reading in the library and a short list of titles recommended for this purpose will be found at the end of the book.

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