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An Introduction to
Electrical Circuit Theory

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Preface

Many textbooks owe their origins to undergraduate lecture courses; this book had its beginnings in a lecture course in engineering science given by the author at the University of Sussex. When the course was begun several years ago the familiar problem of not being able to recommend a single, inexpensive, book to the students taking the course was encountered. The nature of the course structure at Sussex with its major and minor subjects made the problem more difficult because the students attending the course included not only electrical and electronic engineers but undergraduates from many other scientific disciplines too. Educational experiments in the presentation of course material being conducted at the time also meant that printed lecture notes were prepared for the course and it is with these that this book had its beginnings.

While the course contained much circuit theory other topics were included which do not appear here and conversely this book contains several topics not covered in the course. The additional material has been included so that the book may be considered to be an introduction to the subject of circuit theory viewed as a separate discipline and not viewed as a service subject to other disciplines.

The philosophy governing the presentation of the material is that all the circuit laws, methods of analysis and circuit theorems are developed using the simplest possible circuits containing only resistances and d.c. sources. Thus the discussion is not clouded by the examination of the more complicated circuit elements and sources which introduce time variations; the intention is for the student to master the analytical techniques before he goes on to apply them to the more complicated circuits. Application of the techniques to frequency domain circuits is then a logical step which allows the student to concentrate on the frequency domain concepts.

The background knowledge of readers is assumed to be school mathematics and physics but to include no circuit theory. It is also assumed that students will be covering such topics as complex numbers, second-order differential equations and linear algebra simultaneously with their studies of circuit theory and such mathematical topics are not discussed in detail here.

I would like to acknowledge the advice, co-operation and support I have received from my colleagues at the University of Sussex. It is also no cliché to say that this book could not have been created without the active assistance, tolerant

understanding and unflagging encouragement of my wife.

Many of the problems in this book were originally devised by me for Preliminary Year examinations of the University of Sussex and I am grateful to the University of Sussex for permitting me to use them.

G. W.