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CALCULUS MADE EASY:

BEING A VERY-SIMPLEST INTRODUCTION TO
THOSE BEAUTIFUL METHODS OF RECKONING
WHICH ARE GENERALLY CALLED BY THE
TERRIFYING NAMES OF THE

DIFFERENTIAL CALCULUS AND THE INTEGRAL CALCULUS

BY
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THIRD EDITION

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1965

What one fool can do, another can.
(*Ancient Simian Proverb*)

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PUBLISHERS' NOTE ON THE THIRD EDITION

ONLY once in its long and useful life, in 1919, has this book been enlarged and revised. But in twenty-four years much progress can be made, and the methods of 1919 are not likely to be the same as those of 1945. If, therefore, any book is to maintain its usefulness, it is essential that it should be overhauled occasionally so that it may be brought up-to-date where possible, to keep pace with the forward march of scientific development.

For the new edition the book has been reset, and the diagrams modernised. Mr. F. G. W. Brown has been good enough to revise the whole of the book, but he has taken great care not to interfere with the original plan. Thus teachers and students will still recognise their well-known guide to the intricacies of the calculus. While the changes made are not of a major kind, yet their significance may not be inconsiderable. There seems no reason now, even if one ever existed, for excluding from the scope of the text those intensely practical functions, known as the hyperbolic sine, cosine and tangent, whose applications to the methods of integration are so potent and manifold. These have, accordingly, been introduced and applied, with the result that some of the long cumbersome methods of integrating have been displaced, just as a ray of sunshine dispels an obstructing cloud.

The introduction, too, of the very practical integrals :

$$\int e^{pt} \sin kt \cdot dt \quad \text{and} \quad \int e^{pt} \cos kt \cdot dt$$

has eliminated some of the more ancient methods of " Finding Solutions " (Chapter XXI). By their application, shorter and more intelligible ones have grown up naturally instead.

iv PUBLISHERS' NOTE ON THE THIRD EDITION

In the treatment of substitutions, the whole text has been tidied up in order to render it methodically consistent. A few examples have also been added where space permitted, while the whole of the exercises and their answers have been carefully revised, checked and corrected. Duplicated problems have thus been removed and many hints provided in the answers adapted to the newer and more modern methods introduced.

It must, however, be emphatically stated that the plan of the original author remains unchanged ; even in its more modern form, the book still remains a monument to the skill and the courage of the late Professor Silvanus P. Thompson. All that the present reviser has attempted is to revitalize the usefulness of the work by adapting its distinctive utilitarian bias more closely in relation to present-day requirements.

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PROLOGUE

CONSIDERING how many fools can calculate, it is surprising that it should be thought either a difficult or a tedious task for any other fool to learn how to master the same tricks.

Some calculus-tricks are quite easy. Some are enormously difficult. The fools who write the text-books of advanced mathematics—and they are mostly clever fools—seldom take the trouble to show you how easy the easy calculations are. On the contrary, they seem to desire to impress you with their tremendous cleverness by going about it in the most difficult way.

Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

COMMON GREEK LETTERS USED AS SYMBOLS

<i>Capital</i>	<i>Small</i>	<i>English Name</i>	<i>Capital</i>	<i>Small</i>	<i>English Name</i>
<i>A</i>	<i>α</i>	Alpha	<i>Λ</i>	<i>λ</i>	Lambda
<i>B</i>	<i>β</i>	Beta	<i>M</i>	<i>μ</i>	Mu
<i>Γ</i>	<i>γ</i>	Gamma	<i>Ξ</i>	<i>ξ</i>	Xi
<i>Δ</i>	<i>δ</i>	Delta	<i>Π</i>	<i>π</i>	Pi
<i>E</i>	<i>ε</i>	Epsilon	<i>P</i>	<i>ρ</i>	Rho
<i>H</i>	<i>η</i>	Ēta	<i>Σ</i>	<i>σ</i>	Sigma
<i>Θ</i>	<i>θ</i>	Thēta	<i>Φ</i>	<i>φ</i>	Phi
<i>K</i>	<i>κ</i>	Kappa	<i>Ω</i>	<i>ω</i>	Omega