THE CORE OF MATHEMATICS

INTRODUCTORY MONOGRAPHS IN MATHEMATICS

General Editor

A. J. MOAKES, M.A.

Numerical Mathematics

A. J. Moakes

*Exercises in computing with a desk
calculating machine*

Mathematics for Circuits

W. Chellingsworth

The Core of Mathematics

A. J. Moakes

*An introduction to 'modern'
mathematics*

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*An introduction to
'modern' mathematics*

BY

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PREFACE

It is hoped that this short introduction to 'modern' mathematics will be of value to young students who need or who desire more of mathematics than to develop power of manipulation and problem-solving, and to those teachers who have lacked opportunity to come into contact with this approach.

The work is written at a time when, both in the U.S.A. and on the continent of Europe, 'modern' teaching programmes are being widely used and even imposed. In Great Britain and the Commonwealth there is a certain hesitation about this trend, since up to now British students at say 19 have met a wider range of techniques than their opposite numbers elsewhere. In the long run, however, the practical man is better equipped if he really knows what he is doing. Above all, at a time when computing and theoretical physics have changed the outlook of the mathematics user, a modern approach is seen to be not only fundamental but also highly illuminating and thoroughly useful.

Teachers of experimental syllabuses are surprised at the ready reception of a new approach by quite young children: it has come to stay, though we do not yet see in what form or to what extent it will appear in the syllabuses of say ten years hence. This book deals with the fundamentals of the subject as far as they seem likely to affect school courses either directly or indirectly in the years immediately ahead. One would expect that in due course the A-level syllabus in Pure Mathematics would contain much of the material of this book; but, more important, it aims to provide a higher-level treatment of work which in stage-A form is coming into the curriculum of the younger pupil.

Mathematics is the least suitable of all subjects for passive reading; accordingly, at all appropriate points in the text there are exercises, for which solution notes are given at the end of the book.

It is the author's experience that suitable models can provide a great stimulus to enquiry, and a way to deeper understanding. Suggestions for various projects of this sort are made in the text; and details are given in an appendix.

It would be possible to vary the order of study in some minor respects, but the reader is advised to follow the given order as closely as possible, taking 'the rough with the smooth'. It would however be practicable to omit the treatment of ordering in Chapter 4, and to sit

lightly to the derivation of real number in Chapter 5, because one could, at a first reading, base the remaining work entirely on an intuitive appreciation of these concepts.

Chapter 5 is the most difficult in the sense that there appears to be less in the way of graphic or tangible material. This will not be the case for those who have gained insight into the nature of number processes by using a desk machine: such readers will be readily able to appreciate the logic of the integers as presented here.

One is keenly aware that the development of several topics has been cut short at an early stage. Suggestions for further reading are made in the Bibliography and elsewhere.

The author wishes to thank, for their encouragement and criticism, Dr. K. Gruenberg of Queen Mary College, London, Dr. W. Ledermann of the University of Sussex, Dr. G. Matthews of St. Dunstan's College; and also for help in detailed revision, solutions to examples and draughtsmanship his pupils Paul Binding, Steven Groak and John Parker.

St. Paul's School, 1963.

A. J. M.

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GLOSSARY OF NOTATIONS

Phrases: $\forall$ for all; $|$ such that; $\in$ (is) in; $\notin$ (is) not in.

Sets: ϕ the null set;

J the set of all integers; J^+ positive integers; J_0^+ non-negative integers;

R all rational numbers; R^+ positive rationals, etc.; $R^\#$ all *real* numbers.

[Where appropriate the same symbols denote *fields*.]

Laws and relations in sets: only the field symbols $+$, $\times$, $>$ are standard. Others ($*$, ρ , etc.) are defined whenever used.

Affixes ($*$ ' etc.): may have 'local' meanings, but in a given concourse ($\mathcal{C}$), S' means the complement of set S .

p' is the negation of statement p , but some write not- p as $\sim p$.

Laws and relations on sets: $\supset$ includes; $\subset$ is included by; $\cup$ union; $\cap$ intersection; Δ symmetric difference.

Logical connections: $\wedge$ (sometimes $\&$), and; $\vee$, either or both;

$\equiv$, has the same truth-value as ... (i.e. $\Leftrightarrow$);

$\ncong$, has not the same truth value as;

$\Rightarrow$, implies; $\Leftarrow$, is implied by (negated as $\nRightarrow$, etc.).

For elements having a law of composition: i (or e) unique identity element;

x^{-1} unique (pre- and post-) inverse of element x .

For functions, etc.:

h : precedes definition of function denoted by h (e.g. $h: J \rightarrow J, n \rightarrow n^2$);

h^{-1} inverse of function h ;

M^{-1} inverse of matrix M (M' is its transpose);

$|M|$ determinant of (square) matrix M ;

$A \rightarrow B$ (set A) maps into (set B) by rule specified;

$\leftrightarrow$ is in 1-1 correspondence with;

$(J, +, >) \leftrightarrow (S, \times, >)$ denotes correspondence covering the laws and relations shown, i.e. a homomorphism.

In this book Ω has been used for the 3-group of *rotations* of the equilateral triangle, and Δ for the 6-group of its *symmetries*.

We have used cursive capitals for defining conditions. For laws: $\mathcal{A}$ associativity, $\mathcal{C}$ closure, $\mathcal{L}$ Latin-Square $\mathcal{I}_1$ existence of an identity, $\mathcal{I}_2$ existence of unique inverses.

For relations: $\mathcal{D}$ dichotomy, $\mathcal{R}$ reflexivity, $\mathcal{S}$ symmetry, $\mathcal{T}$ transitivity.