Theory of Groundwater Flow

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Theory of Groundwater Flow

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Preface

The aim of this book is to present the fundamentals of the theory of groundwater flow, and the most effective methods for solving groundwater flow problems occurring in civil engineering practice. Technical and practical aspects are not discussed. These will be treated in a companion volume, *Groundwater Recovery and Recharge*, by Professor L. Huisman.

Although the book is in principle intended to be an introductory text, I have not been able to withstand the temptation to include some theoretically rather complicated methods, notably the complex variable method and the finite element method. These are treated in chapters 7, 8 and 10, which may be omitted without impeding the understanding of the remaining parts of the book. Yet I hope that some readers will be attracted by the charm of the complex variable method, or by the power of the finite element method. This latter method has been developed in recent years, mainly for computer calculations, and will certainly become more and more important in the near future, especially when general programmes become available.

None of the methods presented in this book, and practically none of the examples, have been developed by me. Some care has been taken to mention the original sources. In addition to these references, however, I wish to express my personal indebtedness to all those who have contributed to the development of the theory of groundwater flow in the Netherlands, and who can probably best be thanked collectively in the group of engineers together constituting the 'Hydrologic Colloquium'.

I am particularly indebted to Professor G. de Josselin de Jong, who taught me mechanics, and who made me see the power and the beauty of various mathematical techniques for solving engineering problems.

I have much profited from the comments of Dr E. M. Wilson, the editor of this series, on the manuscript in its first stage. Finally I want to thank my sister Alice for producing most of the typescript.

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A.V.

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