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Stability and Stabilization of Nonlinear Systems

Foreword by E. Sontag

Springer

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To Christina, Katerina, and Olympia (I.K.)

To His Family and Friends (Z.P.J.)

Foreword

Control systems and feedback loops are ubiquitous in engineering, in areas such as aerospace control, manufacturing and robotics, active damping, climate control of buildings, process control in chemical plants, electrical power systems, bioengineering, consumer products, and active suspensions, automatic braking systems, and engine timing in the automobile industry. One finds control and feedback in nature as well, for example, in the homeostatic mechanisms that allow organisms to finely tune their internal variables such as temperature, pressure, or chemical levels.

The field of *mathematical control theory* concerns itself with the basic theoretical principles underlying the analysis of feedback and the design of control systems. It differs from the more classical study of dynamical systems in its emphasis on inputs (or controls) and outputs (or measurements). Linearized analysis of systems is the basic foundation of most practical control engineering and has been phenomenally successful. Nonetheless, linearization techniques can only deal with "small" deviations from desired behavior. Thus, the development of tools appropriate to the "global" study of systems with inputs and outputs has been the focus of a major research effort since at least the early 1970s.

In the late 1980s, there emerged a novel paradigm for nonlinear system analysis, based on the notions of "input-to-state stability" and several variants, which allow a seamless integration of classical Lyapunov-like stability theory with input/output operator approaches. The program of research that ensued has as its ultimate goal a complete formulation of the foundations of nonlinear behavior in two dual ways: the analysis of given systems in terms of these notions and the use of these notions in the form of systematic design tools which assign desirable properties to feedback systems. This duality between analysis and design is well summarized by the authors' maxim: "for every method of proving global stability, there is a corresponding method of nonlinear feedback design," and the book systematically applies that principle.

An original and very valuable aspect of this monograph is its treatment not only of ordinary differential equation systems, but also of delay and more general functional differential equations, as well as allowing a study of time-varying systems through "nonuniform" stability notions. Another original feature of the book is that

ISS and its variants are extended to various important classes of interconnected systems using small-gain theorems. This duality between ISS (for single systems) and small-gain (for coupled systems) plays a key role in addressing the problems of robust stability and stabilization.

The field is still active and full of open problems, and this monograph should encourage much further research and further development of applications, many of which are discussed here. The authors of this volume have been two of the main contributors to the program, and in this book they provide a detailed and rigorous introduction, suitable for beginning graduate students as well as for more experienced researchers who wish to transition to the field.

Piscataway, NJ Eduardo Sontag

Preface

Phenomenal progress in nonlinear systems theory has been made during the last decades. It has been reflected in two aspects. On the one hand, internal and external global stability notions have been studied intensely for uncertain nonlinear systems. On the other hand, the applications of these advanced stability results to control engineering systems have led to numerous novel methodologies for the design of nonlinear feedback controllers. It is fair to say that input-to-state stability (ISS), a notion invented by E.D. Sontag in the late 1980s, plays an influential role in the work of many researchers including the authors of this book. ISS has bridged the gap which previously existed between the input–output and the state-space methods, two popular approaches within the control systems community. Roughly speaking, the importance of ISS for the study of nonlinear systems is reflected by the intriguing fact that it captures two main stability notions: Lyapunov stability (i.e., the behavior of the zero-input response with respect to nonzero initial conditions) and input–output stability (i.e., the behavior of the zero-state response with respect to nonzero external inputs).

Nonlinear systems are encountered frequently in almost all branches of science and engineering. In fact, in engineering, physics, economics, and biology, nonlinearity is the rule, and linear systems are rare (which almost exclusively exist only in our computer programs). Despite the importance of nonlinear system theory, graduate students or researchers in mathematics, engineering, physics, economics, and biology often have difficulties in taking advantage of recent advances in mathematical systems and control theories. There are several excellent textbooks that provide nice introductions to nonlinear systems theory, but many recent stability results are scattered in the vast literature. Motivated by this observation, we set our hands to write this monograph about a year and half ago. The specific objectives of this book are described in the following.

The first aim of the book is to provide the basic knowledge needed for a graduate student in order to be able to understand the current research in nonlinear stability theory and nonlinear control theory. A relatively high level of mathematical background is assumed: the reader is required of having basic knowledge in differential equations, calculus, and real analysis. Measure theory is not needed (although measurable functions are met even in the first pages of the book): the reader can replace "measurable" functions by "piecewise continuous" functions. The book is self-contained in the sense that all results are proved in detail by using basic mathematical knowledge and other results presented in the book. Only global stability notions are studied in the present book. It should be mentioned that there are many important results about local or regional stability in the literature.

The second aim of the book is to give a perspective of nonlinear stability theory and nonlinear control theory that is not frequently encountered in the literature. The idea can be stated in the following (informal) way:

"*for every method of proving global stability, there is a corresponding method of nonlinear feedback design*."

Therefore, the book is designed to help the reader to understand this one-to-one correspondence. In the first five chapters the reader is introduced to internal and external stability notions and characterizations. Necessary and sufficient conditions for each stability notion are provided, and a description of the various methods for proving stability is presented. Finally, in Chaps. 6 and 7 of the book the reader is introduced to the various methods of nonlinear feedback design. Each method aims to design a feedback law such that the resulting closed-loop system is "stable." The proof of the stability properties of the closed-loop system is performed by using one particular method of proving stability. We believe that this perspective can help the reader to understand the proposed feedback design methodologies and can inspire the reader to suggest new ones. However, for want of space, some methods of proving stability and feedback design are only briefly mentioned (e.g., the method of using Matrosov's theorem).

The third aim of the book is to show that the same mathematical tools, up to minor modifications, can be used for all kinds of systems. Working within an abstract system-theoretic framework, one can see that systems described by Ordinary Differential Equations (ODEs), systems described by retarded functional differential equations (RFDEs), systems described by coupled retarded functional difference equations and retarded functional differential equations, and sampled-data systems can be analyzed and studied using (almost) the same tools. We believe that this feature is important: many recent contributions to nonlinear systems theory are developed for complex dynamical systems other than those described by ODEs. Furthermore, it has been recognized that feedback laws of new kinds (e.g., feedback laws with delays, hybrid/switching feedback laws) can give rise to features for the closed-loop system that cannot be encountered in systems described by ODEs. In such cases, the closed-loop system in question becomes a system with different mathematical description from the original open-loop system. Time-varying systems are not excluded: the proposed system-theoretic framework can capture all features of time-varying systems (e.g., nonuniform stability phenomena).

As the fourth and last aim of the book, it is the authors' view that nonlinear systems theory has reached a level of maturity which can provide interesting contributions to other areas of applied mathematics. There are many results and examples in the book that illustrate the use of nonlinear systems theory to game theory, fixed-point theory, numerical analysis, and (only superficially) mathematical biology. Some open problems are listed in the last Chap. 8 with a unique objective to entice the reader, in particular graduate students, to develop their novel ideas and techniques, which will contribute to the further development of modern mathematical control theory.

The list of people who must be acknowledged for their support and help to the authors' research is too long to be given. I.K. would like to thank Professor Panagiotis Christofides for his help and good advice that he gave to him when he was a student. Professor John Tsinias has been the academic mentor for Iasson Karafyllis; he owes major gratitude to him that cannot be expressed in a few words. Professors Stelios Kotsios, Eduardo D. Sontag, and Costas Kravaris helped I.K. very much in the first steps of his academic career (each one in a different way). I.K. also owes gratitude to Professor Zhong-Ping Jiang for being one of the first persons who believed in him (and does not mind if it is not right for one author to write acknowledgments for a coauthor!). Z.P.J. would like to thank all his coauthors for contributing to this book, directly or indirectly. Special thanks go to his Ph.D. advisor Laurent Praly for having introduced him to the field of nonlinear control and his two Australian mentors David Hill and Iven Mareels for their long-lasting influence in his career. Z.P.J. also would like to thank Professors A. Isidori, P.V. Kokotović, and E.D. Sontag, and President Jerry Hultin of the Polytechnic Institute of New York University, for the high expectations they set on him, explicitly or implicitly. Yes, it is also a pleasure to thank his nonlinear control friends Jie Huang, Miroslav Krstić, Andy Teel, and Yuan Wang for the moral support over the past years. Finally, the authors would like to thank their students, in particular Yu Jiang, for helping with the typesetting of the book and the drawing of some figures. In the end, all possible errors and typos in the book remain to be the sole responsibility of the authors.

Last but not least, both authors would like to thank their families (Christina, Katerina, Olympia and Xiaoming, Jenny, Jack) for their understanding, support, and patience during the long period when the book was written.

Chania, Greece — Iasson Karafyllis
New York, USA — Zhong-Ping Jiang

Contents

Notations

$\Re$	The set of real numbers
$\Re^+$	The set of nonnegative real numbers
$\Re^n_+$	The set of nth-order vectors of which each component is nonnegative
$\mathbb{N}$	The set of natural numbers
Z	The set of integers
Z^+	The set of nonnegative integers
$\lvert x\rvert$	Euclidean norm of a vector $x \in \Re^n$
x'	The transpose of a vector $x \in \Re^n$
x_{-i}	The vector obtained by deleting the ith component of a vector $x \in \Re^n$, with $n \geq 2$
$\Re^{n\times m}$	The space of real matrices with dimensions $n \times m$
A'	The transpose of a matrix $A \in \Re^{n\times m}$
A^{-1}	The inverse of an invertible square matrix $A \in \Re^{n\times n}$
I	Generic notation of an identity matrix
$\mathrm{int}(A)$	The interior of a set $A \subseteq \Re^n$
K^+	The class of positive continuous functions on $\Re^+$
$\mathcal{N}$	A function $a : \Re^+ \to \Re^+$ is of class $\mathcal{N}$ if a is continuous and nondecreasing with $a(0) = 0$
$\mathrm{Pr}_U(x)$	The projection of $x \in \Re^n$ on a convex set $U \subseteq \Re^n$, i.e., the unique vector y such that $\lvert x - y\rvert = \min_{u\in U} \lvert x - u\rvert$
p.d.	A function $f : A \to \Re^+$, where $A \subseteq \Re^n$ with $0 \in A$, is positive definite (p.d.) if $f(0) = 0$ and $f(x) > 0$ for all $x \in A\backslash\{0\}$
r.u.	A function $f : A \to \Re^+$, where $A \subseteq \Re^n$ is a nonempty and unbounded set, is said to be radially unbounded (r.u.) if, for every $a \geq 0$, the level set $\{x \in A : f(x) \leq a\}$ is bounded
$\mathrm{supp}(f)$	The support of a function $f : A \to \Re$, where $A \subseteq \Re^n$
$x \leq y$	For a pair of vectors $x, y \in \Re^n$, we say that $x \leq y$ if and only if $(y - x) \in \Re^n_+$
$\mathcal{N}_n$	A function $\rho : \Re^n_+ \to \Re^+$ is of class $\mathcal{N}_n$ if ρ is continuous with $\rho(0) = 0$ and such that $\rho(x) \leq \rho(y)$ for all $x, y \in \Re^n_+$ with $x \leq y$

$[V]_{[t_0,t]}$	For $t \geq t_0 \geq 0$, let $[t_0, t] \ni \tau \to V(\tau) = (V_1(\tau), \ldots, V_n(\tau))' \in \Re^n$ be a bounded map. We define $[V]_{[t_0,t]} := (\sup_{\tau\in[t_0,t]} V_1(\tau), \ldots, \sup_{\tau\in[t_0,t]} V_n(\tau))'$
$\Gamma^{(k)}$	We say that $\Gamma : \Re^n_+ \to \Re^m_+$ is nondecreasing if $\Gamma(x) \leq \Gamma(y)$ for all $x, y \in \Re^n_+$ with $x \leq y$. For any positive integer k, we define $\Gamma^{(k)}(x) = \underbrace{\Gamma \circ \Gamma \circ \cdots \circ \Gamma}_{k \text{ times}}(x)$. By convention, we set $\Gamma^{(0)}(x) = x$, for all $x \in \Re^n_+$
1	We define $\mathbf{1} = (1, 1, \ldots, 1)' \in \Re^n$. If $u, v \in \Re$ and $u \leq v$, then $\mathbf{1}u \leq \mathbf{1}v$
K	A function $\alpha : \Re^+ \to \Re^+$ is of class K if α is continuous and increasing with $\alpha(0) = 0$
K_∞	A function $\alpha : \Re^+ \to \Re^+$ is of class K_∞ if it is of class K and satisfies $\lim_{s\to+\infty} \alpha(s) = +\infty$
KL	A function $\beta : \Re^+ \times \Re^+ \to \Re^+$ is of class KL if, for each $t \geq 0$, the mapping $\sigma(\cdot, t)$ is of class K, and, for each $s \geq 0$, the mapping $\beta(s, \cdot)$ is nonincreasing with $\lim_{t\to+\infty} \beta(s, t) = 0$
$\mathcal{E}$	The set of nonnegative continuous functions $\mu : \Re^+ \to \Re^+$ with $\lim_{t\to+\infty} \mu(t) = 0$ and $\int_0^{+\infty} \mu(t)\, dt < +\infty$
$\lVert \cdot \rVert_{\mathcal{X}}$	The norm of the normed linear space $\mathcal{X}$
$B_U[0, r]$	The intersection of $U \subseteq \mathcal{X}$ with the closed sphere of radius $r \geq 0$, centered at $0 \in U$, i.e., $B_U[0, r] := \{u \in U;\ \lVert u\rVert_{\mathcal{X}} \leq r\}$
$\lVert(x, y)\rVert_C$	Unless specified otherwise, the linear space $C = \mathcal{X} \times \mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{Y}$ are normed linear spaces, is endowed with norm $\lVert(x, y)\rVert_C = \sqrt{\lVert x\rVert^2_{\mathcal{X}} + \lVert y\rVert^2_{\mathcal{Y}}}$, for all $(x, y) \in C$
$\mathcal{M}(U)$	The set of all functions $u : \Re^+ \to U$
u_0	The identically zero input, i.e., $u_0(t) = 0 \in U$ for all $t \geq 0$
$\mathcal{L}^\infty(I; A)$	The set of Lebesgue measurable and essentially bounded functions $u : I \to A$ for an interval $I \subseteq \Re$ and a set $A \subseteq \Re^n$
$\mathcal{L}^\infty_{\mathrm{loc}}(I; A)$	The set of Lebesgue measurable and locally essentially bounded functions $u : I \to A$ for an interval $I \subseteq \Re$ and a set $A \subseteq \Re^n$
$\{T_i\}_{i=0}^\infty$	A partition $\pi = \{T_i\}_{i=0}^\infty$ of $\Re^+$ that is an increasing sequence of times with $T_0 = 0$ and $T_i \to +\infty$. Its diameter is defined as $\sup\{T_{i+1} - T_i;\ i = 0, 1, 2, \ldots\}$
$q_\pi(t)$	For every partition $\pi = \{T_i\}_{i=0}^\infty$ of $\Re^+$, $q_\pi(t) := \min\{T \in \pi; t < T\}$
$C^0(A; \Omega)$	The class of continuous functions on a subset A of a normed linear space $\mathcal{X}$, taking values in a subset Ω of a normed linear space $\mathcal{Y}$
$T_r(t)x$	The "r-history" of a function $x : [a - r, b) \to \Re^n$ at time $t \in [a, b)$, for constants $b > a > -\infty$ and $r > 0$. That is, $T_r(t)x$ maps $\theta \in [-r, 0]$ to $x(t + \theta)$
$\lVert x\rVert_r$	For $x : [-r, 0] \to \Re^n$, $\lVert x\rVert_r := \sup_{\theta\in[-r,0]} \lvert x(\theta)\rvert$
q.c.	A function $f : (x, u) \in A \times U \to f(x, u) \in \Re$, where $A \subseteq \mathcal{X}$, $\mathcal{X}$ is a normed linear space and $U \subseteq \Re^m$ is a convex set, is said to be quasi-convex (q.c.) with respect to $u \in U$ if the inequality $f(x, \alpha u_1 + (1 - \alpha)u_2) \leq \max\{f(x, u_1), f(x, u_2)\}$ holds for all $x \in A$, $u_1, u_2 \in U$, and $\alpha \in [0, 1]$

c.c.	A mapping $f : (t, x) \in \mathcal{T} \times A \to f(t, x) \in W$, where $\mathcal{T} = Z^+$ or $\mathcal{T} = \Re^+$, $A \subseteq \mathcal{X}$, and $\mathcal{X}$ and W are normed linear spaces, is said to be completely continuous (c.c.) with respect to $x \in A$, written as $f \in CU(\mathcal{T} \times A; W)$, if for every pair of bounded sets $I \subset \mathcal{T}$ and $S \subseteq A$, f maps $I \times S$ into a bounded set and, additionally, for every $\varepsilon > 0$, there exists $\delta > 0$ such that $\|f(t, x) - f(t, y)\|_W < \varepsilon$ for all $t \in I$ and $x, y \in S$ with $\|x - y\|_{\mathcal{X}} < \delta$
c.l.l.	A mapping $f : (x, d) \in \mathcal{X} \times A \to f(x, d) \in W$, where $A \subseteq \mathcal{Y}$, and $\mathcal{X}$, $\mathcal{Y}$, and W are normed linear spaces, is said to be completely locally Lipschitz (c.l.l.) with respect to $x \in \mathcal{X}$ if, for every pair of bounded sets $S \subset \mathcal{X}$ and $G \subseteq A$, there exists $L \geq 0$ such that $\|f(x, d) - f(y, d)\|_W \leq L\|x - y\|_{\mathcal{X}}$ for all $x, y \in S$ and $d \in G$

Abbreviations

BIC	Boundedness Implies Continuation
RFC	Robust Forward Completeness
ODEs	Ordinary Differential Equations
RFDEs	Retarded Functional Differential Equations
FDEs	Functional Difference Equations
PDEs	Partial Differential Equations
RGAOS	Robust Global Asymptotic Output Stability
URGAOS	Uniform Robust Global Asymptotic Output Stability
RGAS	Robust Global Asymptotic Stability
URGAS	Uniform Robust Global Asymptotic Stability
WIOS	Weighted Input to Output Stability
UWIOS	Uniform Weighted Input to Output Stability
IOS	Input-to-Output Stability
IOpS	Input-to-Output practical Stability
UIOS	Uniform Input-to-Output Stability
WISS	Weighted Input-to-State Stability
UWISS	Uniform Weighted Input to State Stability
ISS	Input-to-State Stability
ISpS	Input-to-State practical Stability
UISS	Uniform Input-to-State Stability
ORCLF	Output Robust Control Lyapunov Functional or Function
SRCLF	State Robust Control Lyapunov Functional or Function
CLF	Control Lyapunov Functional or Function
KSE	Keep Supply at Equilibrium