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Lacra Pavel

Game Theory for Control of Optical Networks

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Preface

Game theory has recently been enjoying increased popularity in the research community, as it provides a new perspective on optimization, networking, and distributed control problems. It incorporates paradigms such as Nash equilibrium and incentive compatibility, it can help quantify individual preferences of decision-makers, and it has an inherently distributed nature. Consequently, game theoretic models have been applied to a variety of problem domains ranging from economics to computer science, networking, and security. Game theoretic models provide not only a basis for analysis but also for design of network protocols and decentralized control schemes. This makes it attractive as a theoretical framework for the design of networks and in particular communication networks. Applications range from power control in wireless communications to congestion control for the Internet, as well as sensor or ad-hoc networks.

As the backbone of the Internet, optical networks provide huge bandwidth and interconnect countries and continents. Unlike conventional networks, which are well-established and even standardized, the optical networks field is much younger, and in fact still evolving. Designing and setting up networking applications within the optical context is inherently more difficult than in conventional wired or wireless networks, in part due to the more complex physical-layer effects, and in part due to the lack of automatic methodologies developed for optical networks. But in these respects, the optical networking field has seen a tremendous growth in recent years.

From a game theory and control perspective, there are a multitude of problems to be tackled in the optical networks area and the field is still in its infancy. Aside from the fact that the area is much younger, research work requires an unusual blend of interdisciplinary expertise. In this spirit, the present monograph draws the author's research background in control theory, practical industrial experience in optical networks, and more than five years of research in game theory and control in optical communications.

The book is focused on mathematical models, methodologies, and game theory for optical networks from a control perspective. The general setup is that, given an optical communication network and some performance measures to be optimized among many networks units/players/channels, one must design an algorithm that achieves as good a performance as possible for each channel. The algorithm should be decentralized and provably convergent, and should use localized and decentralized feedback. By regarding channels in the network as players in a game this multi-objective optimization problem fits within the setup of game theory.

The first theoretical question of interest is how to formulate meaningful and tractable game theoretical problems that can be used as a basis for developing such algorithms, taking into account the various mathematical characteristics arising naturally in optical networks. These mathematical characteristics are imposed by physical constraints and by network specific features and topologies. They translate into constraints on the game formulation (separable versus coupled action space), constraints on player's interaction (localized versus global interaction, i.e., one shot game versus stage or partitioned game), and constraints on players' actions (global constraints versus propagated/modified constraints).

Thus, one of the characteristics of optical networks is that game theoretic formulations and results cannot be transferred directly from other application domains (e.g., wireless networks or congestion control). Due to inherent physical complexities, the optical area uncovers new theoretical problems, such as games with coupled utilities and coupled constraints. Moreover, in the networking setup these constraints are modified across the network links and a framework for dealing with network games has to be developed. This makes game theory in optical networks an excellent starting point that could open new research problems and that could be generalized to other classes of games in networks.

This monograph has a two-fold aim. Its first goal is to provide researchers in the control and game theoretic community with background on the rich problems and the initial results in this area. There is a broad scope for fundamental control and game theoretical research; the hope is that the book will provide background material such that non-specialists in optical networks can approach these research problems within the optical networking domain. The book's second goal is to provide researchers in the networking and optical community with game theoretical methodologies that could be used to solve optical networking problems.

The following topics are covered. In the first part of the book, two chapters present non-cooperative game theory background and some new mathematical results for Nash equilibria computation in games with coupled constraints. In the second part, background and mathematical models for optical networks are presented, followed by game theory formulations developed for various topologies. The basic game considered is a power control game in the class of games with continuous action spaces, coupled utilities in normal form. The book in fact introduces in a systematic, gradual way the different types of game theoretical problems: first games with no coupled constraints in normal form, then games with coupled constraints; all-to-all interaction versus localized player interaction, which leads to games in ladder-nested form; multi-link (single-sink) and finally mesh topologies and how to deal with games in such scenarios, while building on the simpler cases and results. The third part considers issues such as robustness and time-delay effects, as well as other types of game problems in optical networks, including routing and path coloring games.

Much of the work in game theory applications has been within the framework of networking or computer science fields. Essentially, this book uses a mathematical approach developed from a control theoretical perspective. This is different from a networking approach, which is typically application-specific and focused on particular protocols, and unlike algorithmic-only approaches as typically used in computer science. Game theoretical problems are mathematically formulated herein in a systemic manner, with analytic conditions derived for existence and uniqueness of Nash equilibrium. Based on these, iterative algorithms with provable convergence properties to Nash equilibria are developed. The control theoretic approach to games as taken here allows for the treatment of important issues such as stability and time-delay effects in a dynamic system context. The abstract mathematical models and results could be applied to other application domains.

I wish to acknowledge and express my thanks to the many people who influenced and shaped my work over the past several years and thus indirectly helped me in writing this book: Tamer Başar, Tansu Alpcan, Peter Caines, Roland Malhamé, and Eitan Altman from the game theoretical and control community; and Stewart Aitchison, Li Qian, and Dan Kilper from the optical communications community. I am indebted to my graduate students, Yan Pan, Nem Stefanovic, and Quanyan Zhu, with whom many of the research work was done and without whom this book would not have been possible.

Toronto Lacra Pavel

Contents

1 Introduction . . . 1
1.1 Game Theory in Networks . . . 1
1.2 Optical Networks . . . 2
1.3 Scope of This Monograph . . . 5

Part I Game Theory Essentials

2 Basics of Game Theory . . . 11
2.1 Introduction . . . 11
2.2 Game Formulations . . . 12
2.3 Games in Extensive Form . . . 13
2.4 Games in Normal Form . . . 16
2.5 Game Features . . . 17
2.5.1 Strategy Space: Matrix vs. Continuous Games . . . 17
2.5.2 Mixed vs. Pure Strategy Games . . . 18
2.5.3 Competitive Versus Cooperative . . . 18
2.5.4 Repetition . . . 20
2.5.5 Knowledge Information . . . 21
2.6 Solution Concepts . . . 21
2.6.1 Minimax Solution . . . 21
2.6.2 Best Response . . . 22
2.6.3 Nash Equilibrium Solution . . . 23
2.6.4 Pareto Optimality . . . 24
2.7 The Rationality Assumption . . . 25
2.8 Learning in Classical Games . . . 25
2.9 Notes . . . 26

3 Matrix Games . . . 27
3.1 Introduction . . . 27
3.2 Bimatrix Games . . . 27
3.2.1 Mixed Strategies . . . 30
3.3 m-Player Games . . . 32

3.3.1 Pure and Mixed Strategies 32
3.3.2 Mixed-Strategy Cost Functions 33
3.4 Dominance and Best Replies 35
3.4.1 Best-Response Correspondence 36
3.5 Nash Equilibria Theorem 36
3.6 Nash Equilibria Refinements 41
3.6.1 Perfect Equilibrium 41
3.6.2 Proper Equilibrium 42
3.6.3 Strategically Stable Equilibrium 43
3.7 Notes 43

4 Games with Continuous Action Spaces 45
4.1 Introduction 45
4.2 Game Formulations 45
4.3 Extension to Mixed Strategies 46
4.4 Nash Equilibria and Reaction Curves 47
4.5 Existence and Uniqueness Results 49
4.6 Notes 54

5 Computational Results for Games with Coupled Constraints 55
5.1 Introduction 55
5.2 Nash Equilibria and Relaxation via an Augmented Optimization . . 57
5.3 Lagrangian Extension in a Game Setup 60
5.4 Duality Extension 62
5.5 Hierarchical Decomposition in a Game Setup 64
5.6 Notes 67

Part II Game Theory in Optical Networks

6 Optical Networks: Background and Modeling 71
6.1 Introduction 71
6.2 Transmission Basics 74
6.3 Topologies and Setup 75
6.4 Power Control and OSNR Model 78
6.4.1 Point-to-Point Link Model 78
6.4.2 Network Model 86
6.4.3 Link Capacity Constraint 93
6.5 Notes 95

7 Games in Point-to-Point Topologies 97
7.1 Game Formulation 97
7.2 Games Without Coupled Constraints 98
7.2.1 Utility and Nash Equilibrium Solution 98
7.2.2 Iterative Algorithm 104
7.3 Games with Coupled Constraints: Indirect Penalty Approach . . . 106
7.3.1 Nash Equilibrium Solution 107
7.3.2 Iterative Algorithm and Convergence Analysis 110

7.4 Games with Coupled Constraints: Lagrangian Pricing Approach . . 114
7.4.1 Lagrangian Pricing and Duality Extension 114
7.4.2 Iterative Algorithm 119
7.5 Notes 120

8 Games in Network Topologies 121
8.1 Introduction 121
8.2 Games in γ-Link Topologies 122
8.2.1 Partitioned Game Formulation 122
8.3 Games in Single-Sink Topologies 128
8.3.1 Convexity Analysis (Coupled Constraints) 129
8.3.2 Hierarchical Decomposition 130
8.4 Games with Coupled Constraints: Mesh Topologies 134
8.4.1 Partitioned Game Formulation 137
8.4.2 Hierarchical Decomposition 138
8.4.3 Primal–Dual (Channel-Link) Iterative Algorithm 140
8.5 Notes 142

9 Nash Equilibria Efficiency and Numerical Studies 143
9.1 Introduction 143
9.2 A System Optimization Formulation 144
9.3 Distributed Algorithm 148
9.4 Numerical Study of Efficiency in Nash Game 151
9.5 Notes 159

10 Simulations and Experimental Studies 161
10.1 Introduction 161
10.2 Point-to-Point Link Topologies 162
10.2.1 Simulations Results 163
10.2.2 Experimental Results 165
10.3 Multi-link and Mesh Topologies 168
10.3.1 Multi-link Topologies 171
10.3.2 Quasi-ring Topologies 173
10.3.3 Mesh Topologies 177
10.3.4 Experimental Results 180

Part III Robustness, Delay Effects, and Other Problems

11 Robustness and Delay Effects on Network Games 189
11.1 Introduction 189
11.2 System Model 190
11.3 Delay Effects in Network Games Without Constraints 192
11.4 Delay Effects in Network Games with Constraints 197
11.4.1 Delayed Primal–Dual Algorithm 197
11.4.2 Time-Scale Stability Analysis: Point-to-Point Topologies . 201
11.5 Robustness and Delay Effects Combined 204
11.5.1 OSNR Model with Time Delays and Uncertainties 204

11.5.2 Single-Link Analysis 206
11.6 Notes 209

12 Games for Routing and Path Coloring 211
12.1 Introduction 211
12.2 Game-Theoretic Model 213
12.3 Cost Functions and PoA 217
12.4 Solution to Online S-RPC as Nash Equilibria 220
12.5 Notes 223

13 Summary and Conclusions 225

Appendix A Supplementary Material 229
A.1 Notations 229
A.2 Standard Optimization Review 230
A.3 Diagonally Dominant and M-matrices 233
A.4 Maximum Theorem 234
A.5 Fixed-Point Theorems 235
A.6 Projection Theorem 236
A.7 Lipschitz Continuity 236
A.8 Variational Inequalities 237
A.9 Singular Perturbation Theory 237
A.10 Time-Delay Stability Results 240
A.11 Time-Delay Stability Theory 240
A.11.1 Definitions 240
A.11.2 Razumikhin Stability Theory 243

Appendix B List of Notations 247
B.1 Notations and Symbols 247
B.2 List of Symbols 248

References 251

Subject Index 259

List of Acronyms

ABC	Automatic power control
APC	Automatic power control
ASE	Amplified spontaneous emission noise
BER	Bit-error rate
CW	Continuous wave
DSL	Digital subscriber line
GA	Gradient algorithm
LHS	Left-hand side
LS	Light source
NE	Nash equilibrium
OA	Optical amplifier
OADM	Optical add/drop multiplexer
OLT	Optical line terminal
ONTS	Optical network test system
OSA	Optical spectrum analyzer
OSC	Optical service channel
OSNR	Optical signal-to-noise ratio
OXC	Optical cross-connect
PUA	Parallel update algorithm
RHS	Right-hand side
Rx	Receiver
SIR	Signal to interference ratio
Tx	Transmitter
VOA	Variable optical attenuator
VOL	Virtual optical link
WDM	Wavelength division multiplexing