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A Set Theory Workbook

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PREFACE

This book is a companion to *A general topology workbook* published by Birkhäuser last year. In an ideal world the order of publication would have been reversed, for the notation and some of the results of the present book are used in the topology book and on the other hand (the reader may be assured) no topology is used here.

Both books share the word *Workbook* in their titles. They are based on the principle that for at least some branches of mathematics a good way for a student to learn is to be presented with a clear statement of the definitions of the terms with which the subject is concerned and then to be faced with a collection of problems involving the terms just defined. In adopting this approach with my Dundee students of set theory and general topology I found it best not to differentiate too precisely between simple illustrative examples, easy exercises and results which in conventional textbooks would be labelled as Theorems. (I have used the title *Theorem* for some exercises occasionally to label them with their traditional names but mainly to indicate not so much that they are difficult but that they are important and useful.) And, although readers may not find the hints I have provided in this book very generous, the students in my classes got none at all. Not only that, they were not shown the solutions to the exercises until they had made serious attempts to solve them for themselves. That was easy to arrange in the classroom situation but readers of the book can cheat by looking at the full answers provided in Part II. I should like to encourage them not to do that—and to remind them that this is a *Workbook* and that they will derive more benefit from it if they work seriously at the exercises before turning to the solutions.

The titles of Chapters 2 to 13 show that the contents of the book are entirely conventional for an advanced undergraduate or beginning graduate course in set theory. The treatment is intended to be relatively informal, in the sense that definitions and proofs are given for the most part in (hopefully clear) standard “mathematical English” not in logical

symbolism. I hope that the reference in the Introduction to “the first order predicate calculus with equality” will not be found off-putting—it is, after all, only a shorthand way of describing the logical framework that we use as soon as we start to do mathematics seriously.

This is the fourth book in which I have had the help and advice of Nick Dawes with my Latex typesetting; as he breathes a sigh of relief that this is (I think) the last, I want to put on record my gratitude to him for his calm and calming expert assistance. I must again thank the several generations of students in The University of Dundee who wrestled successfully with the material presented here and whose success with it suggested that I turn it into this book. I am very grateful also to my friends—and particularly my wife and daughter—who encouraged me to complete it.

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Dundee, Scotland
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