



# ***Control Engineering***

## ***Series Editor***

William S. Levine  
Department of Electrical and Computer Engineering  
University of Maryland  
College Park, MD  
USA

## ***Editorial Advisory Board***

*Okko Bosgra*  
Delft University  
The Netherlands

*Graham Goodwin*  
University of Newcastle  
Australia

*Iori Hashimoto*  
Kyoto University  
Japan

*Petar Kokotović*  
University of California  
Santa Barbara, CA  
USA

*Manfred Morari*  
ETH  
Zürich  
Switzerland

*William Powers*  
Ford Motor Company (retired)  
Detroit, MI  
USA

*Mark Spong*  
University of Illinois  
Urbana-Champaign  
USA

Yuri Shtessel • Christopher Edwards  
Leonid Fridman • Arie Levant

# Sliding Mode Control and Observation

Y. Shtessel  
Department of Electrical and Computer  
Engineering  
University of Alabama in Huntsville  
Huntsville, AL, USA

C. Edwards  
College of Engineering, Mathematics  
and Physical Science  
University of Exeter  
Exeter, UK

L. Fridman  
Department of Control  
Division of Electrical Engineering  
Faculty of Engineering  
National Autonomous University of Mexico  
Mexico

A. Levant  
Department of Applied Mathematics  
School of Mathematical Sciences  
Tel-Aviv University  
Israel

ISBN 978-0-8176-4892-3      ISBN 978-0-8176-4893-0 (eBook)  
DOI 10.1007/978-0-8176-4893-0  
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2013934106

Mathematics Subject Classifications (2010): 93B12, 93C10, 93B05, 93B07, 93B51, 93B52, 93D25

© Springer Science+Business Media New York 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.birkhauser-science.com](http://www.birkhauser-science.com))

*We dedicate this book with love and  
gratitude to*

*Yuri's wife Nina,  
Chris' parents Shirley and Cyril,  
Leonid's wife Millie,  
Arie's wife Irena.*



# Preface

Control in the presence of uncertainty is one of the main topics of modern control theory. In the formulation of any control problem there is always a discrepancy between the actual plant dynamics and its mathematical model used for the controller design. These discrepancies (or mismatches) mostly come from external disturbances, unknown plant parameters, and parasitic dynamics. Designing control laws that provide the desired closed-loop system performance in the presence of these disturbances/uncertainties is a very challenging task for a control engineer. This has led to intense interest in the development of the so-called robust control methods, which are supposed to solve this problem. In spite of the extensive and successful development of robust adaptive control [159],  $\mathcal{H}_\infty$  control [48], and backstepping [121] techniques, sliding mode control (SMC) remains, probably, the most successful approach in handling bounded uncertainties/disturbances and parasitic dynamics [67, 182, 186].

Historically sliding modes were discovered as a special mode in variable structure systems (VSS). These systems comprise a variety of structures, with rules for switching between structures in real time to achieve suitable system performance, whereas using a single fixed structure could be unstable. The result is VSS, which may be regarded as a combination of subsystems where each subsystem has a fixed control structure and is valid for specified regions of system behavior. It appeared that the closed-loop system may be designed to possess new properties not present in any of the constituent substructures alone. Furthermore, in a special mode, named a sliding mode, these properties include insensitivity to certain (so-called matched) external disturbances and model uncertainties as well as robustness to parasitic dynamics. Achieving reduced-order dynamics of the compensated system in a sliding mode (termed partial dynamical collapse) is also a very important useful property of sliding modes. One of the first books in English to be published on this subject is [85]. The development of these novel ideas began in the Soviet Union in the late 1950s.

The idea of SMC is based on the introduction of a “custom-designed” function, named the sliding variable. As soon as the properly designed sliding variable becomes equal to zero, it defines the sliding manifold (or the sliding surface). The

proper design of the sliding variable yields suitable closed-loop system performance while the system trajectories belong to the sliding manifold. The idea of SMC is to steer the trajectory of the system to the properly chosen sliding manifold and then maintain motion on the manifold thereafter by means of control, thus exploiting the main features of the sliding mode: its insensitivity to external and internal disturbances matched by the control, ultimate accuracy, and finite-time convergence of the sliding variables to zero.

The first well-cited text in English on SMC was by Itkis and published in 1976 [113]. By 1980, the main contributions in SMC theory had been completed and subsequently reported in Utkin's 1981 monograph (in Russian) and its subsequent English version [182]. A comprehensive review was published by DeCarlo et al. in [56]. In these publications (see also the advanced results presented in the later works [186] and [67]), the two-step procedure for SMC design was clearly stated.

The first step involves the design of a switching function so that the system motion on the sliding manifold (termed the sliding motion) satisfies the design specifications. The second step is concerned with the selection of a control law, which will make the sliding manifold attractive to the system state in the presence of external and internal disturbances/uncertainties. Note that this control law is not necessarily discontinuous.

SMC-based observers allow estimation of the system states in the presence of unknown external disturbances, which can also be explicitly reconstructed online by an observer.

Control chattering still remained a problem impeding SMC implementation. Addressing control chattering was the main motivation for the emerging so-called second-order sliding. Thus, the already matured conventional SMC theory received a significant boost in the middle of the 1980s: when new "second-order" ideas appeared [132] and then, in the beginning of 2000s, when "higher-order" [124] concepts were introduced. The introduction of these new paradigms was dictated by the following reasons:

1. The conventional sliding mode design approach requires the system relative degree to be equal to one with respect to the sliding variable. This can seriously constrain the choice of the sliding variable.
2. Also, very often, a sliding mode controller yields high-frequency switching control action that leads to the so-called chattering effect, which is difficult to avoid or attenuate.

These intrinsic difficulties of conventional SMC are mitigated by higher-order sliding mode (HOSM) controllers that are able to drive to zero not only the sliding variable but also its  $k - 1$  successive derivatives ( $k$ th-order sliding mode). The novel approach is effective for arbitrary relative degrees, and the well-known chattering effect is significantly reduced, since the high-frequency control switching is "hidden" in the higher derivative of the sliding variable.

When implemented in discrete time, HOSM provides sliding accuracy proportional to the  $k$ th power of the sampling time, which makes HOSM an enhanced-accuracy robust control technique. Since only the  $k$ th derivative of the sliding manifold is proportional to the high-frequency switching control signal, the switch-



ing amplitude is well attenuated at the sliding manifold level, which significantly reduces chattering.

The unique power of the approach is revealed by the development of practical arbitrary-order real-time robust exact differentiators, whose performance is proved to be asymptotically optimal in the presence of Lebesgue-measurable input noises. The HOSM differentiators are used in advanced HOSM-based observers for the estimation of the system state in the presence of unknown external disturbances, which are also reconstructed online by the observers. In addition HOSM-based parameter observers have been developed as well.

The combination of a HOSM controller with the above-mentioned HOSM-based differentiator produces a robust and exact output-feedback controller. No detailed mathematical models of the plant are needed. SMC of arbitrary smoothness can be achieved by artificially increasing the relative degree of the system, significantly attenuating the chattering effect. For instance, the continuous control function can be obtained if virtual control in terms of the control derivative is designed in terms of SMC. In this case, the control function will be continuous, since it is equal to the integral of the high-frequency switching function. In the case of parasitic/unmodeled dynamics the SMC function will switch with lower frequency (the control chattering). Designing the SMC in terms of the derivative of the control function yields chattering attenuation.

The practicality of conventional SMC and HOSM control and observation techniques is demonstrated by a large variety of applications that include DC/DC and AC/DC power converters, control of AC and DC motors and generators, aircraft and missile guidance and control, and robot control.

SMC is a mature theory. This textbook is mostly based on the class notes for the graduate-level courses on SMC and Nonlinear Control that have been taught at the Department of Electrical and Computer Engineering, the University of Alabama in Huntsville; at the Department of Engineering, the University of Leicester; at the Department of Control Engineering and Robotics, the Engineering Faculty, the National Autonomous University of Mexico and at the Department of Applied Mathematics, the Tel Aviv University for the last 10–15 years. The course notes have been constantly updated during these years to include newly developed HOSM control and observation techniques.

This textbook provides the reader with a broad range of material from first principles up to the current state of the art in the area of SMC and observation presented in a pedagogical fashion. As such it is appropriate for graduate students with a basic knowledge of classical control theory and some knowledge of state-space methods and nonlinear systems. The resulting design procedures are emphasized using Matlab/Simulink software.

Fully worked out design examples are an additional feature. Practical case studies, which present the results of real sliding mode controller implementations, are used to illustrate the successful practical application of the theory. Each chapter is equipped with exercises for homework assignments.

The textbook is structured as follows.

In Chap. 1 we “intuitively” introduce the main concepts of SMC for regulation and tracking problems, as well as state and input observation using only basic control system theory. The sliding variable and SMC design techniques are demonstrated on tutorial examples and graphical expositions. The reaching and sliding phases of the compensated system dynamics are identified. Advanced concepts associated with conventional sliding modes, including sliding mode observers/differentiators and second-order sliding mode controllers, are studied on a tutorial level. Robust output tracking controller design based on a relative degree approach is studied. The design framework comprises conventional and second-order sliding modes as well as sliding mode observers. The main advantages of SMC and HOSM control, including robustness, finite-time convergence, and reduced-order compensated dynamics, are demonstrated through numerous examples and simulation plots.

In Chap. 2 we formulate and rigorously study the conventional multivariable SMC problem using linear algebra and Lyapunov function techniques. The interpretation of the sliding surface design problem as a straightforward linear state-feedback problem for a particular subsystem is emphasized. A variety of methods for sliding surface design, including linear quadratic minimization and eigenvalue placement algorithms, are presented. Possible control design strategies to enforce a sliding motion, including the unit-vector control structure, are described, and the problem of smoothing undesirable discontinuous signals is addressed. The output-feedback SMC techniques that do not require measurement of the system states are presented. Integral sliding modes (ISM) that are a special type of conventional SMC are discussed in detail. The ability of ISM to be initiated without a reaching phase is emphasized. The specific property of ISM that consists of retaining the order of the compensated system is studied. The use of ISM for disturbance compensation is discussed together with a linear quadratic regulation (LQR) problem, “robustified” via ISM. Several examples illustrate the ISM concept.

In Chap. 3 a detailed coverage of conventional sliding mode observers (CSMOs) for state estimation and unknown input reconstruction in dynamic systems is presented. The design techniques for a variety of CSMOs are rigorously studied using linear algebra and Lyapunov function techniques. The robustness properties of CSMO are discussed. Several examples illustrate the CSMO design and demonstrate their performance via simulations. The chapter ends with a list of exercises for homework assignments.

In Chap. 4 second-order sliding mode (2-sliding mode or 2-SM) control is studied as a new generation of conventional SMC. The main definitions, properties and design frameworks for 2-SM control, and associated observers/differentiators are rigorously presented. The essential properties of 2-SM control, including finite-time convergence to zero of the sliding variable and its derivative in the presence of disturbances/uncertainties as well as the ability of computer-implemented 2-SM control to provide enhanced stabilization accuracy that is proportional to the square of the time increment, are emphasized. Several particular types of 2-SM control algorithms, including twisting and super-twisting controllers, the suboptimal

control algorithm, the control algorithm with prescribed convergence law, and the quasi-continuous control algorithm, are introduced. A special case of 2-SM, super-twisting SMC with variable gains, is also studied analytically and experimentally. An output regulation problem solution is described in terms of the above-mentioned 2-SM controllers. In particular the chattering attenuation capabilities of 2-SM controllers are emphasized. Numerous examples illustrate the advantages in terms of performance of 2-SM controllers. The chapter culminates with a list of exercises for homework assignments.

In Chap. 5 we study a very important robustness property of conventional SMC and 2-SM-based controllers to parasitic dynamics using frequency-domain techniques. The describing function technique is used to estimate both amplitude and frequency of the switching control oscillation as soon as the transient response is over. The robustness of conventional SMC to first- and second-order parasitic dynamics is described. The analysis of oscillations with finite amplitude and frequency in 2-SM controllers, including the twisting and super-twisting controllers and the quasi-optimal controller, in the presence of first- and second-order parasitic dynamics, is performed. Numerous examples illustrate the performances of conventional SMC and 2-SM-based controllers. Exercises for homework assignment are presented at the end of the chapter.

In Chap. 6 the concept of 2-SM control is generalized by introducing HOSM control that is a new generation of SMC. The ability of HOSM control to drive the sliding variable and its  $k - 1$  successive derivatives (a so-called  $k$ th-order sliding mode) to zero in finite time is rigorously derived and discussed. Two families of HOSM control algorithms, a nested SMC algorithm and a quasi-continuous control algorithm, are introduced. Homogeneity and contractivity-based techniques that are used for HOSM control analysis and design are described. The efficacy of HOSM control for systems with arbitrary relative degree with respect to the sliding variable is identified. Significant attenuation of the well-known chattering effect via HOSM control is described. The HOSM-based arbitrary-order online robust exact differentiator is introduced and discussed. Several examples are presented to illustrate the performance of HOSM controllers and differentiators. The application of the HOSM controllers and differentiators to blood glucose regulation, using an insulin pump in feedback, illustrates the HOSM algorithms. A list of exercises for homework assignments completes the chapter.

In Chap. 7 we revisit the state observation and identification problem, previously studied in Chap. 3. In this chapter state observation, identification, and input reconstruction are discussed using algorithms based on HOSM exact differentiators. HOSM observers for nonlinear systems are described. Parameter identification algorithms using HOSM techniques are presented and discussed. Several examples, including pendulum and satellite dynamics estimation and identification, are presented to illustrate the performance of the HOSM observation and identification algorithms. The exercises for homework assignments are presented at the end of the chapter.

In Chap. 8 we describe output regulation and tracking problems addressed by conventional SMC and HOSM controllers driven by sliding mode disturbance

observers (SMC–SMDO). The particular features of the application of SMC/HOSM observers to the above-mentioned output regulation/tracking problems, including the necessity to differentiate the measured output in order to implement the SMC or HOSM controller, as well as the possibility of reconstructing unknown external disturbances via SMC/HOSM observers with the possibility to compensate for them within a traditional continuous controller, are emphasized. The continuous SMC–SMDO design techniques are illustrated with two case studies: launch vehicle attitude control and satellite formation control. A variety of exercises are presented at the end of this chapter to facilitate homework assignments.

The contribution of the authors to this textbook is as follows: Dr. Shtessel has written the Preface and Chaps. 1 and 8 and has contributed to Chap. 5 by writing Sect. 5.2 and Chap. 6 by writing Sect. 6.11. Dr. Edwards has written Chaps. 2 and 3. He has also carried out the bulk of the editorial work. Dr. Fridman has written Chaps. 5 and 7 and has contributed to Chap. 2 by writing Sect. 2.7 and to Chaps. 4 and 6 by writing Sects. 4.7, 4.8, and 6.11. Dr. Levant has written Chaps. 4 and 6.

The authors would also like to acknowledge the graduate and postdoctoral students of the Department of Automatic Control, the National Autonomous University of Mexico, Francisco Bejarano, Jorge Davila, Lizet Fragueta, Ana Gabriela Gallardo, Tenoch Gonzalez, Antonio Rosales, and Carlos Vazquez, for their invaluable help in preparing examples and exercises. We would also like to thank our colleagues, Professors Igor Boiko, Leonid Freidovich, Elio Usai, and Vadim Utkin for their careful reading of early drafts of the manuscript and for their constructive criticisms and suggestions for improvement.

Huntsville, USA  
Exeter, UK  
Mexico, Mexico  
Tel Aviv, Israel

Y. Shtessel  
C. Edwards  
L. Fridman  
A. Levant

# Contents

<b>1</b>	<b>Introduction: Intuitive Theory of Sliding Mode Control</b> .....	1
1.1	Main Concepts of Sliding Mode Control .....	3
1.2	Chattering Avoidance: Attenuation and Elimination .....	9
1.2.1	Chattering Elimination: Quasi-Sliding Mode .....	9
1.2.2	Chattering Attenuation: Asymptotic Sliding Mode .....	11
1.3	Concept of Equivalent Control .....	17
1.4	Sliding Mode Equations .....	18
1.5	The Matching Condition and Insensitivity Properties .....	19
1.6	Sliding Mode Observer/Differentiator .....	20
1.7	Second-Order Sliding Mode .....	23
1.8	Output Tracking: Relative Degree Approach .....	27
1.8.1	Conventional Sliding Mode Controller Design .....	28
1.8.2	Integral Sliding Mode Controller Design .....	30
1.8.3	Super-Twisting Controller Design .....	33
1.8.4	Prescribed Convergence Law Controller Design .....	36
1.9	Notes and References .....	40
1.10	Exercises .....	41
<b>2</b>	<b>Conventional Sliding Modes</b> .....	43
2.1	Introduction .....	43
2.1.1	Filippov Solution .....	44
2.1.2	Concept of Equivalent Control .....	47
2.2	State-Feedback Sliding Surface Design .....	50
2.2.1	Regular Form .....	53
2.2.2	Eigenvalue Placement .....	55
2.2.3	Quadratic Minimization .....	58
2.3	State-Feedback Relay Control Law Design .....	61
2.3.1	Single-Input Nominal Systems .....	61
2.3.2	Single-Input Perturbed Systems .....	62
2.3.3	Relay Control for Multi-input Systems .....	67

2.4	State-Feedback Unit-Vector Control .....	68
2.4.1	Design in the Presence of Matched Uncertainty .....	68
2.4.2	Design in the Presence of Unmatched Uncertainty .....	71
2.5	Output Tracking with Integral Action .....	75
2.6	Output-Based Hyperplane Design .....	77
2.6.1	Static Output-Feedback Hyperplane Design .....	78
2.6.2	Static Output-Feedback Control Law Development.....	83
2.6.3	Dynamic Output-Feedback Hyperplane Design .....	85
2.6.4	Dynamic Output-Feedback Control Law Development ....	87
2.6.5	Case Study: Vehicle Stability in a Split-Mu Maneuver ....	88
2.7	Integral Sliding Mode Control .....	89
2.7.1	Problem Formulation .....	90
2.7.2	Control Design Objective.....	91
2.7.3	Linear Case.....	91
2.7.4	ISM Compensation of Unmatched Disturbances .....	94
2.8	Notes and References.....	96
2.9	Exercises .....	99
<b>3</b>	<b>Conventional Sliding Mode Observers .....</b>	<b>105</b>
3.1	Introduction .....	105
3.2	A Simple Sliding Mode Observer .....	106
3.3	Robustness Properties of Sliding Mode Observers .....	111
3.4	A Generic Conventional Sliding Mode Observer .....	121
3.5	A Sliding Mode Observer for Nonlinear Systems .....	128
3.6	Fault Detection: A Simulation Example.....	133
3.7	Notes and References.....	136
3.8	Exercises .....	137
<b>4</b>	<b>Second-Order Sliding Mode Controllers and Differentiators.....</b>	<b>143</b>
4.1	Introduction .....	143
4.2	2-Sliding Mode Controllers .....	147
4.2.1	Twisting Controller .....	148
4.2.2	Suboptimal Algorithm .....	151
4.2.3	Control Algorithm with Prescribed Convergence Law .....	152
4.2.4	Quasi-Continuous Control Algorithm .....	153
4.2.5	Accuracy of 2-Sliding Mode Controllers .....	155
4.3	Control of Relative Degree One Systems.....	155
4.3.1	Super-Twisting Controller .....	155
4.3.2	First-Order Differentiator.....	159
4.4	Differentiator-Based Output-Feedback 2-SM Control.....	161
4.5	Chattering Attenuation .....	163
4.6	Case Study: Pendulum Control .....	166
4.6.1	Discontinuous Control.....	167
4.6.2	Chattering Attenuation.....	169

4.7	Variable-Gain Super-Twisting Control .....	170
4.7.1	Problem Statement .....	171
4.7.2	The Variable-Gain Super-Twisting Algorithm .....	172
4.8	Case Study: The Mass–Spring–Damper System .....	176
4.8.1	Model Description .....	176
4.8.2	Problem Statement .....	177
4.8.3	Control Design .....	178
4.8.4	Experimental Results .....	179
4.9	Notes and References .....	179
4.10	Exercises .....	182
<b>5</b>	<b>Analysis of Sliding Mode Controllers in the Frequency Domain .....</b>	<b>183</b>
5.1	Introduction .....	183
5.2	Conventional SMC Algorithm: DF Analysis .....	184
5.3	Twisting Algorithm: DF Analysis .....	193
5.4	Super-Twisting Algorithm: DF Analysis .....	196
5.4.1	DF of Super-Twisting Algorithm .....	196
5.4.2	Existence of the Periodic Solutions .....	198
5.4.3	Stability of Periodic Solution .....	200
5.5	Prescribed Convergence Control Law: DF Analysis .....	201
5.6	Suboptimal Algorithm: DF Analysis .....	203
5.7	Comparisons of 2-Sliding Mode Control Algorithms .....	205
5.8	Notes and References .....	208
5.9	Exercises .....	208
<b>6</b>	<b>Higher-Order Sliding Mode Controllers and Differentiators .....</b>	<b>213</b>
6.1	Introduction .....	214
6.2	Single-Input Single-Output Regulation Problem .....	216
6.3	Homogeneity, Finite-Time Stability, and Accuracy .....	217
6.4	Homogeneous Sliding Modes .....	222
6.5	Accuracy of Homogeneous 2-Sliding Modes .....	223
6.6	Arbitrary-Order Sliding Mode Controllers .....	225
6.6.1	Nested Sliding Controllers .....	225
6.6.2	Quasi-continuous Sliding Controllers .....	227
6.7	Arbitrary-Order Robust Exact Differentiation .....	228
6.8	Output-Feedback Control .....	230
6.9	Tuning of the Controllers .....	233
6.9.1	Control Magnitude Tuning .....	233
6.9.2	Parametric Tuning .....	233
6.10	Case Study: Car Steering Control .....	234
6.11	Case Study: Blood Glucose Regulation .....	237
6.11.1	Introduction to Diabetes .....	237
6.11.2	Insulin–Glucose Regulation Dynamical Model .....	240
6.11.3	Higher-Order Sliding Mode Controller Design .....	241
6.11.4	Simulation .....	244

6.12	Notes and References .....	247
6.13	Exercises .....	248
<b>7</b>	<b>Observation and Identification via HOSM Observers .....</b>	<b>251</b>
7.1	Observation/Identification of Mechanical Systems .....	252
7.1.1	Super-Twisting Observer .....	253
7.1.2	Equivalent Output Injection Analysis .....	255
7.1.3	Parameter Identification .....	259
7.2	Observation in Single-Output Linear Systems .....	265
7.2.1	Non-perturbed Case .....	265
7.2.2	Perturbed Case .....	266
7.2.3	Design of the Observer for Strongly Observable Systems .....	268
7.3	Observers for Single-Output Nonlinear Systems.....	274
7.3.1	Differentiator-Based Observer .....	275
7.3.2	Disturbance Identification .....	278
7.4	Regulation and Tracking Controllers Driven by SM Observers .....	280
7.4.1	Motivation .....	280
7.4.2	Problem Statement .....	281
7.4.3	Theoretically Exact Output-Feedback Stabilization (EOFS) .....	282
7.4.4	Output Integral Sliding Mode Control .....	283
7.4.5	Precision of the Observation and Identification Processes .....	284
7.5	Notes and References.....	286
7.6	Exercises .....	286
<b>8</b>	<b>Disturbance Observer Based Control: Aerospace Applications .....</b>	<b>291</b>
8.1	Problem Formulation .....	291
8.1.1	Asymptotic Compensated Dynamics .....	292
8.1.2	Finite-Time-Convergent Compensated Dynamics.....	293
8.1.3	Sliding Variable Disturbed Dynamics.....	294
8.1.4	Output Tracking Error Disturbed Dynamics.....	294
8.2	Perturbation Term Reconstruction via a Disturbance Observer .....	295
8.2.1	SMDO Based on Conventional SMC .....	295
8.2.2	SMDO Based on Super-Twisting Control .....	296
8.2.3	Design of the SMC Driven by the SMDO .....	297
8.3	Case Study: Reusable Launch Vehicle Control .....	298
8.3.1	Mathematical Model of Reusable Launch Vehicle .....	298
8.3.2	Reusable Launch Vehicle Control Problem Formulation ..	300
8.3.3	Multiple-Loop Asymptotic SMC/SMDO Design .....	301
8.3.4	Flight Simulation Results and Analysis.....	305



8.4	Case Study: Satellite Formation Control .....	309
8.4.1	Satellite Formation Mathematical Model .....	310
8.4.2	Satellite Formation Control in SMC/SMDO .....	313
8.5	Simulation Study .....	314
8.6	Notes and References .....	316
8.7	Exercises .....	318
<b>A</b>	<b>Mathematical Preliminaries</b> .....	<b>321</b>
A.1	Linear Algebra .....	321
A.1.1	Rank and Determinant .....	321
A.1.2	Eigenvalues and Eigenvectors .....	322
A.1.3	QR Decomposition .....	323
A.1.4	Norms .....	323
A.1.5	Quadratic Forms .....	324
<b>B</b>	<b>Describing Functions</b> .....	<b>327</b>
B.1	Describing Function Fundamentals .....	327
B.1.1	Low-Pass Filter Hypothesis and Describing Function .....	328
B.1.2	Limit Cycle Analysis Using Describing Functions .....	328
B.1.3	Stability Analysis of the Limit Cycle .....	329
<b>C</b>	<b>Linear Systems Theory</b> .....	<b>331</b>
C.1	Introduction .....	331
C.1.1	Linear Time-Invariant Systems .....	331
C.1.2	Controllability and Observability .....	332
C.1.3	Invariant Zeros .....	333
C.1.4	State Feedback Control .....	334
C.1.5	Static Output Feedback Control .....	335
<b>D</b>	<b>Lyapunov Stability</b> .....	<b>337</b>
D.1	Local Results .....	338
D.2	Global Results .....	338
D.2.1	Quadratic Stability .....	339
	<b>Bibliography</b> .....	<b>343</b>
	<b>Index</b> .....	<b>353</b>