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Sliding Mode Control and Observation

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We dedicate this book with love and gratitude to
Yuri's wife Nina,
Chris' parents Shirley and Cyril,
Leonid's wife Millie,
Arie's wife Irena.

Preface

Control in the presence of uncertainty is one of the main topics of modern control theory. In the formulation of any control problem there is always a discrepancy between the actual plant dynamics and its mathematical model used for the controller design. These discrepancies (or mismatches) mostly come from external disturbances, unknown plant parameters, and parasitic dynamics. Designing control laws that provide the desired closed-loop system performance in the presence of these disturbances/uncertainties is a very challenging task for a control engineer. This has led to intense interest in the development of the so-called robust control methods, which are supposed to solve this problem. In spite of the extensive and successful development of robust adaptive control [159], $\mathcal{H}_\infty$ control [48], and backstepping [121] techniques, sliding mode control (SMC) remains, probably, the most successful approach in handling bounded uncertainties/disturbances and parasitic dynamics [67, 182, 186].

Historically sliding modes were discovered as a special mode in variable structure systems (VSS). These systems comprise a variety of structures, with rules for switching between structures in real time to achieve suitable system performance, whereas using a single fixed structure could be unstable. The result is VSS, which may be regarded as a combination of subsystems where each subsystem has a fixed control structure and is valid for specified regions of system behavior. It appeared that the closed-loop system may be designed to possess new properties not present in any of the constituent substructures alone. Furthermore, in a special mode, named a sliding mode, these properties include insensitivity to certain (so-called matched) external disturbances and model uncertainties as well as robustness to parasitic dynamics. Achieving reduced-order dynamics of the compensated system in a sliding mode (termed partial dynamical collapse) is also a very important useful property of sliding modes. One of the first books in English to be published on this subject is [85]. The development of these novel ideas began in the Soviet Union in the late 1950s.

The idea of SMC is based on the introduction of a “custom-designed” function, named the sliding variable. As soon as the properly designed sliding variable becomes equal to zero, it defines the sliding manifold (or the sliding surface). The proper design of the sliding variable yields suitable closed-loop system performance while the system trajectories belong to the sliding manifold. The idea of SMC is to steer the trajectory of the system to the properly chosen sliding manifold and then maintain motion on the manifold thereafter by means of control, thus exploiting the main features of the sliding mode: its insensitivity to external and internal disturbances matched by the control, ultimate accuracy, and finite-time convergence of the sliding variables to zero.

The first well-cited text in English on SMC was by Itkis and published in 1976 [113]. By 1980, the main contributions in SMC theory had been completed and subsequently reported in Utkin's 1981 monograph (in Russian) and its subsequent English version [182]. A comprehensive review was published by DeCarlo et al. in [56]. In these publications (see also the advanced results presented in the later works [186] and [67]), the two-step procedure for SMC design was clearly stated.

The first step involves the design of a switching function so that the system motion on the sliding manifold (termed the sliding motion) satisfies the design specifications. The second step is concerned with the selection of a control law, which will make the sliding manifold attractive to the system state in the presence of external and internal disturbances/uncertainties. Note that this control law is not necessarily discontinuous.

SMC-based observers allow estimation of the systemstates in the presence of unknown external disturbances, which can also be explicitly reconstructed online by an observer.

Control chattering still remained a problem impeding SMC implementation. Addressing control chattering was the main motivation for the emerging so-called second-order sliding. Thus, the already matured conventional SMC theory received a significant boost in the middle of the 1980s: when new "second-order" ideas appeared [132] and then, in the beginning of 2000s, when "higher-order" [124] concepts were introduced. The introduction of these new paradigms was dictated by the following reasons:

1. The conventional sliding mode design approach requires the system relative degree to be equal to one with respect to the sliding variable. This can seriously constrain the choice of the sliding variable.
2. Also, very often, a sliding mode controller yields high-frequency switching control action that leads to the so-called chattering effect, which is difficult to avoid or attenuate.

These intrinsic difficulties of conventional SMC are mitigated by higher-order sliding mode (HOSM) controllers that are able to drive to zero not only the sliding variable but also its $k-1$ successive derivatives (kth-order sliding mode). The novel approach is effective for arbitrary relative degrees, and the well-known chattering effect is significantly reduced, since the high-frequency control switching is "hidden" in the higher derivative of the sliding variable.

When implemented in discrete time, HOSM provides sliding accuracy proportional to the kth power of the sampling time, which makes HOSM an enhanced-accuracy robust control technique. Since only the kth derivative of the sliding manifold is proportional to the high-frequency switching control signal, the switching amplitude is well attenuated at the sliding manifold level, which significantly reduces chattering.

The unique power of the approach is revealed by the development of practical arbitrary-order real-time robust exact differentiators, whose performance is proved to be asymptotically optimal in the presence of Lebesgue-measurable input noises. The HOSM differentiators are used in advanced HOSM-based observers for the estimation of the system state in the presence of unknown external disturbances, which are also reconstructed online by the observers. In addition HOSM-based parameter observers have been developed as well.

The combination of a HOSM controller with the above-mentioned HOSM-based differentiator produces a robust and exact output-feedback controller. No detailed mathematical models of the plant are needed. SMC of arbitrary smoothness can be achieved by artificially increasing the relative degree of the system, significantly attenuating the chattering effect. For instance, the continuous control function can be obtained if virtual control in terms of the control derivative is designed in terms of SMC. In this case, the control function will be continuous, since it is equal to the integral of the high-frequency switching function. In the case of parasitic/unmodeled dynamics the SMC function will switch with lower frequency (the control chattering). Designing the SMC in terms of the derivative of the control function yields chattering attenuation.

The practicality of conventional SMC and HOSM control and observation techniques is demonstrated by a large variety of applications that include DC/DC and AC/DC power converters, control of AC and DC motors and generators, aircraft and missile guidance and control, and robot control.

SMC is a mature theory. This textbook is mostly based on the class notes for the graduate-level courses on SMC and Nonlinear Control that have been taught at the Department of Electrical and Computer Engineering, the University of Alabama in Huntsville; at the Department of Engineering, the University of Leicester; at the Department of Control Engineering and Robotics, the Engineering Faculty, the National Autonomous University of Mexico and at the Department of Applied Mathematics, the Tel Aviv University for the last 10–15 years. The course notes have been constantly updated during these years to include newly developed HOSM control and observation techniques.

This textbook provides the reader with a broad range of material from first principles up to the current state of the art in the area of SMC and observation presented in a pedagogical fashion. As such it is appropriate for graduate students with a basic knowledge of classical control theory and some knowledge of state-space methods and nonlinear systems. The resulting design procedures are emphasized using Matlab/Simulink software.

Fully worked out design examples are an additional feature. Practical case studies, which present the results of real sliding mode controller implementations, are used to illustrate the successful practical application of the theory. Each chapter is equipped with exercises for homework assignments.

The textbook is structured as follows.

In Chap. 1 we “intuitively” introduce the main concepts of SMC for regulation and tracking problems, as well as state and input observation using only basic control system theory. The sliding variable and SMC design techniques are demonstrated on tutorial examples and graphical expositions. The reaching and sliding phases of the compensated system dynamics are identified. Advanced concepts associated with conventional sliding modes, including sliding mode observers/differentiators and second-order sliding mode controllers, are studied on a tutorial level. Robust output tracking controller design based on a relative degree approach is studied. The design framework comprises conventional and second-order sliding modes as well as sliding mode observers. The main advantages of SMC and HOSM control, including robustness, finite-time convergence, and reduced-order compensated dynamics, are demonstrated through numerous examples and simulation plots.

In Chap. 2 we formulate and rigorously study the conventional multivariable SMC problem using linear algebra and Lyapunov function techniques. The interpretation of the sliding surface design problem as a straightforward linear state-feedback problem for a particular subsystem is emphasized. A variety of methods for sliding surface design, including linear quadratic minimization and eigenvalue placement algorithms, are presented. Possible control design strategies to enforce a sliding motion, including the unit-vector control structure, are described, and the problem of smoothing undesirable discontinuous signals is addressed. The output-feedback SMC techniques that do not require measurement of the system states are presented. Integral sliding modes (ISM) that are a special type of conventional SMC are discussed in detail. The ability of ISM to be initiated without a reaching phase is emphasized. The specific property of ISM that consists of retaining the order of the compensated system is studied. The use of ISM for disturbance compensation is discussed together with a linear quadratic regulation (LQR) problem, “robustified” via ISM. Several examples illustrate the ISM concept.

In Chap. 3 a detailed coverage of conventional sliding mode observers (CSMOs) for state estimation and unknown input reconstruction in dynamic systems is presented. The design techniques for a variety of CSMOs are rigorously studied using linear algebra and Lyapunov function techniques. The robustness properties of CSMO are discussed. Several examples illustrate the CSMO design and demonstrate their performance via simulations. The chapter ends with a list of exercises for homework assignments.

In Chap. 4 second-order sliding mode (2-sliding mode or 2-SM) control is studied as a new generation of conventional SMC. The main definitions, properties and design frameworks for 2-SM control, and associated observers/differentiators are rigorously presented. The essential properties of 2-SM control, including finite-time convergence to zero of the sliding variable and its derivative in the presence of disturbances/uncertainties as well as the ability of computer-implemented 2-SM control to provide enhanced stabilization accuracy that is proportional to the square of the time increment, are emphasized. Several particular types of 2-SM control algorithms, including twisting and super-twisting controllers, the suboptimal control algorithm, the control algorithm with prescribed convergence law, and the quasi-continuous control algorithm, are introduced. A special case of 2-SM, super-twisting SMC with variable gains, is also studied analytically and experimentally. An output regulation problem solution is described in terms of the above-mentioned 2-SM controllers. In particular the chattering attenuation capabilities of 2-SM controllers are emphasized. Numerous examples illustrate the advantages in terms of performance of 2-SM controllers. The chapter culminates with a list of exercises for homework assignments.

In Chap. 5 we study a very important robustness property of conventional SMC and 2-SM-based controllers to parasitic dynamics using frequency-domain techniques.The describing function technique is used to estimate both amplitude and frequency of the switching control oscillation as soon as the transient response is over. The robustness of conventional SMC to first- and second-order parasitic dynamics is described. The analysis of oscillations with finite amplitude and frequency in 2-SM controllers, including the twisting and super-twisting controllers and the quasi-optimal controller, in the presence of first- and second-order parasitic dynamics, is performed. Numerous examples illustrate the performances of conventional SMC and 2-SM-based controllers. Exercises for homework assignment are presented at the end of the chapter.

In Chap. 6 the concept of 2-SM control is generalized by introducing HOSM control that is a new generation of SMC. The ability of HOSM control to drive the sliding variable and its $k-1$ successive derivatives (a so-called kth-order sliding mode) to zero in finite time is rigorously derived and discussed. Two families of HOSM control algorithms, a nested SMC algorithm and a quasi-continuous control algorithm, are introduced. Homogeneity and contractivity-based techniques that are used for HOSM control analysis and design are described. The efficacy of HOSM control for systems with arbitrary relative degree with respect to the sliding variable is identified. Significant attenuation of the well-known chattering effect via HOSM control is described. The HOSM-based arbitrary-order online robust exact differentiator is introduced and discussed. Several examples are presented to illustrate the performance of HOSM controllers and differentiators. The application of the HOSM controllers and differentiators to blood glucose regulation, using an insulin pump in feedback, illustrates the HOSM algorithms. A list of exercises for homework assignments completes the chapter.

In Chap. 7 we revisit the state observation and identification problem, previously studied in Chap. 3. In this chapter state observation, identification, and input reconstruction are discussed using algorithms based on HOSM exact differentiators. HOSM observers for nonlinear systems are described. Parameter identification algorithms using HOSM techniques are presented and discussed. Several examples, including pendulum and satellite dynamics estimation and identification, are presented to illustrate the performance of the HOSM observation and identification algorithms. The exercises for homework assignments are presented at the end of the chapter.

In Chap. 8 we describe output regulation and tracking problems addressed by conventional SMC and HOSM controllers driven by sliding mode disturbance observers (SMC–SMDO). The particular features of the application of SMC/HOSM observers to the above-mentioned output regulation/tracking problems, including the necessity to differentiate the measured output in order to implement the SMC or HOSM controller, as well as the possibility of reconstructing unknown external disturbances via SMC/HOSM observers with the possibility to compensate for them within a traditional continuous controller, are emphasized. The continuous SMC–SMDO design techniques are illustrated with two case studies: launch vehicle attitude control and satellite formation control. A variety of exercises are presented at the end of this chapter to facilitate homework assignments.

The contribution of the authors to this textbook is as follows: Dr. Shtessel has written the Preface and Chaps. 1 and 8 and has contributed to Chap. 5 by writing Sect. 5.2 and Chap. 6 by writing Sect. 6.11. Dr. Edwards has written Chaps. 2 and 3. He has also carried out the bulk of the editorial work. Dr. Fridman has written Chaps. 5 and 7 and has contributed to Chap. 2 by writing Sect. 2.7 and to Chaps. 4 and 6 by writing Sects. 4.7, 4.8, and 6.11. Dr. Levant has written Chaps. 4 and 6.

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