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Geometric Optics

*Theory and Design of Astronomical Optical
Systems Using Mathematica[®]*

Birkhäuser
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Preface

A very wide selection of excellent books are available to the reader interested in geometric optics. Roughly speaking, these texts can be divided into three main classes.

In the first class (see, for instance, [1]–[9]), we find books that present the theoretical aspects of the subject, usually starting from the Lagrangian and Hamiltonian formulations of geometric optics. These texts analyze the relations between geometric optics, mechanics, partial differential equations, and the wave theory of optics. The second class comprises books that focus on the applications of this theory to optical instruments. In these books, some essential formulae (which are reported without providing proofs) are used to propose exact or approximate solutions to real-world problems (an excellent example of this class is represented by [10]). The third class contains books that approach the subject in a manner that is intermediate between the first two classes (see, for instance, [11]–[16]).

The aim of this book, which could be placed in the third class, is *to provide the reader with the mathematical background needed to design many optical combinations that are used in astronomical telescopes and cameras*.¹ The results presented here were obtained by using a different approach to third-order aberration theory as well as the extensive use of the software package *Mathematica*®.

The different approach to third-order aberration theory adopted in this book is based on Fermat's principle and on the use of particular optical paths (not rays) termed *stigmatic paths*. This approach makes it easy to derive the third-order aberration formulae. In this way, the reader is able to understand and handle the formulae required to design optical combinations without resorting to the much more complex Hamiltonian formalism and Seidel's relations. On the other hand, the Hamiltonian formalism has unquestionable theoretical utility considering its important applications in

¹For a good example of a professional textbook on astronomical optics, see [13].

optics, in mechanics, and in the theory of partial differential equations. For this reason, Hamiltonian optics is widely discussed in Chapters 9–11.

The use of *Mathematica*[®] to design optical combinations is shown to be very convenient. In fact, although the aberration formulae are obtained in an elementary way, their application in the design process necessitates a lot of calculations. Using *Mathematica*[®], it is possible to implement programs that allow us to realize the third-order design of all the astronomical combinations described in this book. Although experience has shown that a design based on third-order optics is not always acceptable, this approach can be used as a starting point for any optimization method available in professional software, such as *OSLO* and *ATMOS*, the simplest versions of which can be freely downloaded from the Internet. However, we must bear in mind that optimization methods will only give correct results if the data used in the approximate design are very similar to those used in the final project. These methods must be handled with great care, since they will very often lead to a new design that is worse than the original one. The reason for this is that the function to be minimized contains many minima that are very close to each other and do not correspond to an effective improvement in the optical combination. For this reason, the author, with the help of A. Limongiello, developed the software *Optisoft* (which runs in the Microsoft Windows environment), which allows the *final* forms of all the optical combinations considered in this book to be obtained.

In the first chapter, the essential aspects of an optical system \mathbb{S} with an axis of rotational symmetry are introduced. Moreover, we analyze all of the data supplied by optical software in order to *check* whether a given optical system \mathbb{S} is acceptable or not. Chapter 2 describes the Gaussian characteristics of \mathbb{S} : conjugate planes, magnification, focal and nodal points, principal planes and optical invariants. The matrix form of the Gaussian approximation is also presented in detail. All of the Gaussian data for an optical system can be derived using the notebook *GaussianData*.²

In Chapter 3, a new approach to third-order monochromatic aberration that is based on both Fermat's principle and *stigmatic paths* is described. Here it is shown that these optical paths can be used in Fermat's principle instead of the real rays, with the advantage that the stigmatic paths are completely known, since they are determined by Gaussian optics. The third-order aberrations for any optical system can be obtained in mathematical form using the notebook *TotalAberrations*. It should be noted that the symbolic formulae are so dense for optical systems containing many elements with finite thicknesses that they are not practical to apply.

²A program written with *Mathematica*[®] can be saved as a notebook or a package.

Chapter 4 contains an analysis of Newtonian and Cassegrain telescopes based on conical mirrors. In Chapters 5 and 6 we study photographic cameras containing lenses and mirrors (Schmidt, Wright, Houghton, and Maksutov cameras), as well as the corresponding catadioptric Cassegrain telescopes. Finally, the third-order design of achromatic doublets or apochromatic doublets and triplets is discussed in Chapter 7. Some other interesting optical devices, including the Klevtsov combination and the Baker–Schmidt flat-field camera, are studied in Chapter 8.

Finally, the Lagrangian and Hamiltonian formulations for geometric optics and Seidel’s third-order aberration theory are treated in Chapters 9–11.

Each optical combination analyzed in this book is accompanied by a notebook that automates its third-order design. All of these notebooks work in versions 4, 5, and 6 of *Mathematica*[®] and may be downloaded from the Publisher’s website at: <http://www.birkhauser.com/978-0-8176-4871-8>. These notebooks represent an integral part of the book for many reasons. First, they contain many calculations that appear in the book and many worked exercises. Moreover, many other exercises can be carried out by the reader him- or herself. Finally, carefully studying the programs contained in the notebooks could provide a useful way for readers to learn how to program with *Mathematica*[®].

We conclude by noting that amateurs with sufficient knowledge of mathematics may find it interesting to learn how to derive the formulae listed in many manuals from the general laws of geometric optics. On the other hand, amateurs who are not interested in learning the mathematical background of optics can use the notebooks contained in the book to rapidly obtain the third-order designs of many cameras and telescopes used in astronomy.

Naples, Italy
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