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Differentiable Manifolds
Second Edition

Lawrence Conlon

Reprint of the 2001 Second Edition

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This book is dedicated to my wife Jackie, with much love

Contents

Preface to the Second Edition

In revising this book for a second edition, I have added a significant amount of new material, dropping the subtitle "A first course". It is hoped that this will make the book more useful as a reference while still allowing it to be used as the basis of a first course on differentiable manifolds. In such a course, one should omit some or all of the material marked with an asterisk. More information about these optional topics will be given below.

Presupposed is a good grounding in general topology and modern algebra, especially linear algebra and the analogous theory of modules over a commutative, unitary ring. Mastery of the central topics of this book should prepare students for advanced courses and seminars in differential topology and geometry.

There are certain basic themes of which the student should be aware. The first concerns the role of differentiation as a process of linear approximation of nonlinear problems. The well–understood methods of linear algebra are then applied to the resulting linear problem and, where possible, the results are reinterpreted in terms of the original nonlinear problem. The process of solving differential equations (*i.e.*, integration) is the reverse of differentiation. It reassembles an infinite array of linear approximations, resulting from differentiation, into the original nonlinear data. This is the principal tool for the reinterpretation of the linear algebra results referred to above.

It is expected that the student has been exposed to the above processes in the setting of Euclidean spaces, at least in low dimensions. This is what we will refer to as *local calculus*, characterized by explicit computations in a fixed coordinate system. The concept of a "differentiable manifold" provides the setting for *global calculus*, characterized (where possible) by coordinate-free procedures. Where (as is often the case) coordinate-free procedures are not feasible, we will be forced to use local coordinates that vary from region to region of the manifold. When theorems are proven in this way, it becomes necessary to show independence of the choice of coordinates. The way in which these local reference frames fit together globally can be extremely complicated, giving rise to problems of a *topological* nature. In the global theory, geometric topology and, sometimes, algebraic topology become essential features.

These themes of *linearization*, *(re)integration*, and *global versus local* will be emphasized repeatedly.

Although a certain familiarity with the local theory is presupposed, we will try to reformulate that theory in a more organized and conceptual way that will make it easier to treat the global theory. Thus, this book will incorporate a modern treatment of the elements of multivariable calculus.

Fundamental to the global theory of differentiable manifolds is the concept of a *vector bundle*. As the global theory is developed, the tangent bundle, the cotangent bundle and various tensor bundles will play increasingly important roles, as will the related notions of infinitesimal G-structures and integrable G-structures.

For conceptual simplicity, all manifolds, functions, bundles, vector fields, Lie groups, homogeneous spaces, *etc.*, will be smooth of class C^∞. It is possible to adapt the treatment to smoothness of class C^k, $1 \leq k < \infty$, but the technical problems that arise are distracting and the usefulness of this level of generality is limited. On the other hand, in much of the literature, the study of Lie groups and homogeneous spaces is carried out in the real analytic (C^ω) category. In these treatments, it is customary to note that C^∞ groups can be proven to be analytic, hence that no generality is lost. It seems to the author, however, that nothing would be gained by this approach and that the ideal of keeping this book as self-contained as possible would be compromised.

The optional topics (sections, subsections and one chapter, with titles terminating in an asterisk) can safely be omitted without creating serious gaps in the overall presentation. One topic that is new to this edition, covering spaces and the fundamental group, is not starred and should not be omitted unless the students have seen it in some prior course.

Some of the optional topics fall into subgroupings, any one of which can be included without dependence on the others. Thus, Subsection 2.9.B and Sections 3.10 and 4.2 constitute a brief introduction to Morse theory, one of the most useful tools in differential topology. Similarly, Sections 3.9, 6.5, and 8.7 constitute an introduction to degree theory, together with some classical topological applications, but in this case any one of these three sections can be treated without serious logical dependence on the others. Apart from minor revisions, this treatment of degree theory is not new to this edition. In Subsection 1.6.B, we classify 1-manifolds. This intuitively plausible result needed is only in the optional Section 3.9. Also easily omitted is the brief Subsection 1.6.A, this being an extended remark on cobordism theory.

New to this edition is an optional introductory treatment of Whitney's imbedding theorems (Subsection 3.7.C). We prove only the "easy" Whitney theorem, while stating carefully the general theorem. Imbeddings of manifolds in Euclidean space will be used only in treating some other optional topics, namely, the smoothing of continuous maps and homotopies (Subsection 3.8.B) and the existence of Morse functions (Section 3.10).

In Chapter 5, an introduction to Lie theory, adequate for a first course on manifolds, requires only the first two sections. Accordingly, Sections 5.3 (the closed subgroup theorem and related topics) and 5.4 (homogeneous spaces) are optional.

Certain topics in de Rham theory, Sections 8.8 (Poincaré duality) and 8.9 (a version of the de Rham theorem), can be omitted, as can the treatment of foliations defined by closed 1-forms (Section 9.3). Also easily omitted is the brief treatment of Riemannian homogeneous and symmetric spaces (Section 10.7). Finally, Chapter 11, on principal bundles and their role in geometry, gathers together and slightly expands on topics treated in various parts of the first edition and can be reserved to introduce a more advanced course or seminar.

There are some significant changes in the appendices also. The original Appendix A has been replaced by one that gives the construction of the universal covering space. The former Appendix D (Sard's theorem) has been moved to the main body of the text. The current Appendix D (formerly Appendix E) has been expanded to include a proof of the de Rham theorem for singular as well as Čech cohomology.

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I am grateful to the late Robby Gardner and his students at Chapel Hill who "beta tested" eight chapters of a preliminary version of the first edition of my book in an intensive, one-semester graduate course. Their many suggestions were most helpful in the final revisions. Others whose input was helpful include Geoffrey Mess, Gary Jensen, Alberto Candel, Nicola Arcozzi and Tony Nielsen. I particularly want to thank Filippo De Mari, whose beautiful class notes, written when he was one of my students in an earlier version of this course, were immensely useful in subsequent revisions and first suggested to me the idea of writing a book. Finally, my students in the academic year 1999–2000 have offered much helpful input toward the final version of this edition.