

Job #: 111368

Author Name: Conlon

Title of Book: Differentiable Manifolds

ISBN #: 9780817647667



Modern Birkhäuser Classics

Many of the original research and survey monographs in pure and applied mathematics published by Birkhäuser in recent decades have been groundbreaking and have come to be regarded as foundational to the subject. Through the MBC Series, a select number of these modern classics, entirely uncorrected, are being re-released in paperback (and as eBooks) to ensure that these treasures remain accessible to new generations of students, scholars, and researchers.

Differentiable Manifolds

Second Edition

Lawrence Conlon

Reprint of the 2001 Second Edition

Birkhäuser
Boston • Basel • Berlin

Lawrence Conlon
Department of Mathematics
Washington University
St. Louis, MO 63130-4899
U.S.A.

Originally published in the series *Birkhäuser Advanced Texts*

ISBN-13: 978-0-8176-4766-7

e-ISBN-13: 978-0-8176-4767-4

DOI: 10.1007/978-0-8176-4767-4

Library of Congress Control Number: 2007940493

Mathematics Subject Classification (2000): 57R19, 57R22, 57R25, 57R30, 57R45, 57R35, 57R55, 53A05, 53B05, 53B20, 53C05, 53C10, 53C15, 53C22, 53C29, 22E15

©2008 Birkhäuser Boston

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Birkhäuser Boston, c/o Springer Science+Business Media LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Cover design by Alex Gerasev.

Printed on acid-free paper.

9 8 7 6 5 4 3 2 1

www.birkhauser.com

Lawrence Conlon

Differentiable Manifolds

Second Edition

Birkhäuser
Boston • Basel • Berlin

Lawrence Conlon
Department of Mathematics
Washington University
St. Louis, MO 63130-4899
U.S.A.

Library of Congress Cataloging-in-Publication Data

Conlon, Lawrence, 1933-
Differentiable manifolds / Lawrence Conlon.—2nd ed.
p. cm.— (Birkhäuser advanced texts)
Includes bibliographical references and index.
ISBN 0-8176-4134-3 (alk. paper)—ISBN 3-7643-4134-3 (alk. paper)
I. Differentiable manifolds. I. Title. II. Series.
QA614.3.C66 2001
516.6—dc21

2001025140

AMS Subject Classifications: 57R19, 57R22, 57R25, 57R30, 57R35, 57R45, 57R50, 57R55, 53A05,
53B05, 53B20, 53C05, 53C10, 53C15, 53C22, 53C29, 22E15

Printed on acid-free paper.
©2001 Birkhäuser Boston, 2nd Edition
©1993 Birkhäuser Boston, 1st Edition

Birkhäuser 

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Birkhäuser Boston, c/o Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

ISBN 0-8176-4134-3 SPIN 10722989
ISBN 3-7643-4134-3

Typeset by the author in L^AT_EX.
Printed and bound by Hamilton Printing Company, Rensselaer, NY.
Printed in the United States of America.

9 8 7 6 5 4 3 2 1

This book is dedicated to my wife Jackie, with much love

Contents

Preface to the Second Edition	xi
Acknowledgments	xiii
Chapter 1. Topological Manifolds	1
1.1. Locally Euclidean Spaces	1
1.2. Topological Manifolds	3
1.3. Quotient Constructions and 2-Manifolds	6
1.4. Partitions of Unity	17
1.5. Imbeddings and Immersions	20
1.6. Manifolds with Boundary	22
1.7. Covering Spaces and the Fundamental Group	26
Chapter 2. The Local Theory of Smooth Functions	41
2.1. Differentiability Classes	41
2.2. Tangent Vectors	42
2.3. Smooth Maps and their Differentials	50
2.4. Diffeomorphisms and Maps of Constant Rank	54
2.5. Smooth Submanifolds of Euclidean Space	58
2.6. Constructions of Smooth Functions	62
2.7. Smooth Vector Fields	65
2.8. Local Flows	71
2.9. Critical Points and Critical Values	80
Chapter 3. The Global Theory of Smooth Functions	87
3.1. Smooth Manifolds and Mappings	87
3.2. Diffeomorphic Structures	93
3.3. The Tangent Bundle	94
3.4. Cocycles and Geometric Structures	98
3.5. Global Constructions of Smooth Functions	104
3.6. Smooth Manifolds with Boundary	107
3.7. Smooth Submanifolds	110
3.8. Smooth Homotopy and Smooth Approximations	116
3.9. Degree Theory Modulo 2*	119
3.10. Morse Functions*	124
Chapter 4. Flows and Foliations	131
4.1. Complete Vector Fields	131
4.2. The Gradient Flow and Morse Functions*	136
4.3. The Lie Bracket	142
4.4. Commuting Flows	145
4.5. Foliations	150

Chapter 5. Lie Groups and Lie Algebras	161
5.1. Basic Definitions and Facts	161
5.2. Lie Subgroups and Subalgebras	170
5.3. Closed Subgroups*	173
5.4. Homogeneous Spaces*	178
Chapter 6. Covectors and 1-Forms	183
6.1. Dual Bundles	183
6.2. The space of 1-forms	185
6.3. Line Integrals	190
6.4. The First Cohomology Space	195
6.5. Degree Theory on S^1 *	202
Chapter 7. Multilinear Algebra and Tensors	209
7.1. Tensor Algebra	209
7.2. Exterior Algebra	217
7.3. Symmetric Algebra	226
7.4. Multilinear Bundle Theory	227
7.5. The Module of Sections	230
Chapter 8. Integration of Forms and de Rham Cohomology	239
8.1. The Exterior Derivative	239
8.2. Stokes' Theorem and Singular Homology	245
8.3. The Poincaré Lemma	258
8.4. Exact Sequences	264
8.5. Mayer–Vietoris Sequences	267
8.6. Computations of Cohomology	271
8.7. Degree Theory*	274
8.8. Poincaré Duality*	276
8.9. The de Rham Theorem*	281
Chapter 9. Forms and Foliations	289
9.1. The Frobenius Theorem Revisited	289
9.2. The Normal Bundle and Transversality	293
9.3. Closed, Nonsingular 1-forms*	296
Chapter 10. Riemannian Geometry	303
10.1. Connections	304
10.2. Riemannian Manifolds	311
10.3. Gauss Curvature	315
10.4. Complete Riemannian Manifolds	322
10.5. Geodesic Convexity	334
10.6. The Cartan Structure Equations	337
10.7. Riemannian Homogeneous Spaces*	342
Chapter 11. Principal Bundles*	347
11.1. The Frame Bundle	347
11.2. Principal G -Bundles	351
11.3. Cocycles and Reductions	354
11.4. Frame Bundles and the Equations of Structure	357

Appendix A. Construction of the Universal Covering	369
Appendix B. The Inverse Function Theorem	373
Appendix C. Ordinary Differential Equations	379
C.1. Existence and uniqueness of solutions	379
C.2. A digression concerning Banach spaces	382
C.3. Smooth dependence on initial conditions	383
C.4. The Linear Case	385
Appendix D. The de Rham Cohomology Theorem	387
D.1. Čech cohomology	387
D.2. The de Rham–Čech complex	391
D.3. Singular Cohomology	397
Bibliography	403
Index	405

Preface to the Second Edition

In revising this book for a second edition, I have added a significant amount of new material, dropping the subtitle “A first course”. It is hoped that this will make the book more useful as a reference while still allowing it to be used as the basis of a first course on differentiable manifolds. In such a course, one should omit some or all of the material marked with an asterisk. More information about these optional topics will be given below.

Presupposed is a good grounding in general topology and modern algebra, especially linear algebra and the analogous theory of modules over a commutative, unitary ring. Mastery of the central topics of this book should prepare students for advanced courses and seminars in differential topology and geometry.

There are certain basic themes of which the student should be aware. The first concerns the role of differentiation as a process of linear approximation of nonlinear problems. The well-understood methods of linear algebra are then applied to the resulting linear problem and, where possible, the results are reinterpreted in terms of the original nonlinear problem. The process of solving differential equations (*i.e.*, integration) is the reverse of differentiation. It reassembles an infinite array of linear approximations, resulting from differentiation, into the original nonlinear data. This is the principal tool for the reinterpretation of the linear algebra results referred to above.

It is expected that the student has been exposed to the above processes in the setting of Euclidean spaces, at least in low dimensions. This is what we will refer to as *local calculus*, characterized by explicit computations in a fixed coordinate system. The concept of a “differentiable manifold” provides the setting for *global calculus*, characterized (where possible) by coordinate-free procedures. Where (as is often the case) coordinate-free procedures are not feasible, we will be forced to use local coordinates that vary from region to region of the manifold. When theorems are proven in this way, it becomes necessary to show independence of the choice of coordinates. The way in which these local reference frames fit together globally can be extremely complicated, giving rise to problems of a *topological* nature. In the global theory, geometric topology and, sometimes, algebraic topology become essential features.

These themes of *linearization*, *(re)integration*, and *global versus local* will be emphasized repeatedly.

Although a certain familiarity with the local theory is presupposed, we will try to reformulate that theory in a more organized and conceptual way that will make it easier to treat the global theory. Thus, this book will incorporate a modern treatment of the elements of multivariable calculus.

Fundamental to the global theory of differentiable manifolds is the concept of a *vector bundle*. As the global theory is developed, the tangent bundle, the cotangent

bundle and various tensor bundles will play increasingly important roles, as will the related notions of infinitesimal G -structures and integrable G -structures.

For conceptual simplicity, all manifolds, functions, bundles, vector fields, Lie groups, homogeneous spaces, *etc.*, will be smooth of class C^∞ . It is possible to adapt the treatment to smoothness of class C^k , $1 \leq k < \infty$, but the technical problems that arise are distracting and the usefulness of this level of generality is limited. On the other hand, in much of the literature, the study of Lie groups and homogeneous spaces is carried out in the real analytic (C^ω) category. In these treatments, it is customary to note that C^∞ groups can be proven to be analytic, hence that no generality is lost. It seems to the author, however, that nothing would be gained by this approach and that the ideal of keeping this book as self-contained as possible would be compromised.

The optional topics (sections, subsections and one chapter, with titles terminating in an asterisk) can safely be omitted without creating serious gaps in the overall presentation. One topic that is new to this edition, covering spaces and the fundamental group, is not starred and should not be omitted unless the students have seen it in some prior course.

Some of the optional topics fall into subgroupings, any one of which can be included without dependence on the others. Thus, Subsection 2.9.B and Sections 3.10 and 4.2 constitute a brief introduction to Morse theory, one of the most useful tools in differential topology. Similarly, Sections 3.9, 6.5, and 8.7 constitute an introduction to degree theory, together with some classical topological applications, but in this case any one of these three sections can be treated without serious logical dependence on the others. Apart from minor revisions, this treatment of degree theory is not new to this edition. In Subsection 1.6.B, we classify 1-manifolds. This intuitively plausible result needed is only in the optional Section 3.9. Also easily omitted is the brief Subsection 1.6.A, this being an extended remark on cobordism theory.

New to this edition is an optional introductory treatment of Whitney's imbedding theorems (Subsection 3.7.C). We prove only the "easy" Whitney theorem, while stating carefully the general theorem. Imbeddings of manifolds in Euclidean space will be used only in treating some other optional topics, namely, the smoothing of continuous maps and homotopies (Subsection 3.8.B) and the existence of Morse functions (Section 3.10).

In Chapter 5, an introduction to Lie theory, adequate for a first course on manifolds, requires only the first two sections. Accordingly, Sections 5.3 (the closed subgroup theorem and related topics) and 5.4 (homogeneous spaces) are optional.

Certain topics in de Rham theory, Sections 8.8 (Poincaré duality) and 8.9 (a version of the de Rham theorem), can be omitted, as can the treatment of foliations defined by closed 1-forms (Section 9.3). Also easily omitted is the brief treatment of Riemannian homogeneous and symmetric spaces (Section 10.7). Finally, Chapter 11, on principal bundles and their role in geometry, gathers together and slightly expands on topics treated in various parts of the first edition and can be reserved to introduce a more advanced course or seminar.

There are some significant changes in the appendices also. The original Appendix A has been replaced by one that gives the construction of the universal covering space. The former Appendix D (Sard's theorem) has been moved to the main body of the text. The current Appendix D (formerly Appendix E) has been expanded to include a proof of the de Rham theorem for singular as well as Čech cohomology.

Acknowledgments

I am grateful to the late Robby Gardner and his students at Chapel Hill who “beta tested” eight chapters of a preliminary version of the first edition of my book in an intensive, one-semester graduate course. Their many suggestions were most helpful in the final revisions. Others whose input was helpful include Geoffrey Mess, Gary Jensen, Alberto Candel, Nicola Arcozzi and Tony Nielsen. I particularly want to thank Filippo De Mari, whose beautiful class notes, written when he was one of my students in an earlier version of this course, were immensely useful in subsequent revisions and first suggested to me the idea of writing a book. Finally, my students in the academic year 1999–2000 have offered much helpful input toward the final version of this edition.