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Algebra and Analysis

for Engineers and Scientists

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PREFACE

This book evolved from a one-year sequence of courses offered by the authors at Iowa State University. The audience for this book typically included theoretically oriented first- or second-year graduate students in various engineering or science disciplines. Subsequently, while serving as Chair of the Department of Electrical Engineering, and later, as Dean of the College of Engineering at the University of Notre Dame, the first author continued using this book in courses aimed primarily at graduate students in control systems. Since administrative demands precluded the possibility of regularly scheduled classes, the Socratic method was used in guiding students in self study. This method of course delivery turned out to be very effective and satisfying to student and teacher alike. Feedback from colleagues and students suggests that this book has been used in a similar manner elsewhere.

The original objectives in writing this book were to provide the reader with appropriate mathematical background for graduate study in engineering or science; to provide the reader with appropriate prerequisites for more advanced subjects in mathematics; to allow the student in engineering or science to become familiar with a great deal of pertinent mathematics in a rapid and efficient manner without sacrificing rigor; to give the reader a unified overview of applicable mathematics, thus enabling him or her to choose additional courses in mathematics more intelligently; and to make it possible for the student to understand at an early stage of his or her graduate studies the mathematics used in the current literature (e.g., journal articles, monographs, and the like).

Whereas the objectives enumerated above for writing this book were certainly pertinent over twenty years ago, they are even more compelling today. The reasons for this are twofold. First, today's graduate students in engineering or science are expected to be more knowledgeable and sophisticated in mathematics than students in the past. Second, today's graduate students in engineering or science are expected to be familiar with a *great deal* of ancillary material (primarily in the computer science area), acquired in courses that did not even exist a couple of decades ago. In view of these added demands on the students' time, to become familiar with a great deal of mathematics in an efficient manner, without sacrificing rigor, seems essential.

Since the original publication of this book, progress in technology, and consequently, in applications of mathematics in engineering and science, has been phenomenal. However, it must be emphasized that the type of mathematics *itself* that is being utilized in these applications did not experience corresponding substantial changes. This is particularly the case for algebra and analysis at the intermediate level, as addressed in the present book. Accordingly, the material of the present book is as current today as it was at the time when this book first appeared. (*Plus ça change, plus c'est la même chose.*—Alphonse Karr, 1849.)

This book may be viewed as consisting essentially of three parts: *set theory* (Chapter 1), *algebra* (Chapters 2–4), and *analysis* (Chapters 5–7). Chapter 1 is a prerequisite for all subsequent chapters. Chapter 2 emphasizes *abstract algebra* (semigroups, groups, rings, etc.) and may essentially be skipped by those who are not interested in this topic. Chapter 3, which addresses *linear spaces and linear transformations*, is a prerequisite for Chapters 4, 6, and 7. Chapter 4, which treats *finite-dimensional vector spaces and linear transformations on such spaces* (matrices) is required for Chapters 6 and 7. In Chapter 5, *metric spaces* are treated. This chapter is a prerequisite for the subsequent chapters. Finally, Chapters 6 and 7 consider *Banach and Hilbert spaces and linear operators on such spaces*, respectively.

The choice of *applications* in a book of this kind is subjective and will always be susceptible to criticisms. We have attempted to include applications of algebra and analysis that have broad appeal. These applications, which may be omitted without loss of continuity, are presented at the ends of Chapters 2, 4, 5, 6, and 7 and include topics dealing with *ordinary differential equations*, *integral equations*, *applications of the contraction mapping principle*, *minimization of functionals*, *an example from optimal control*, and *estimation of random variables*.

All exercises are an integral part of the text and are given when they arise, rather than at the end of a chapter. Their intent is to further the reader's understanding of the subject matter on hand.

The prerequisites for this book include the usual background in undergraduate mathematics offered to students in engineering or in the sciences at universities in the United States. Thus, in addition to graduate students, this book is suitable for advanced senior undergraduate students as well, and for self study by practitioners.

Concerning the labeling of items in the book, some comments are in order. Sections are assigned numerals that reflect the chapter and the section numbers. For example, Section 2.3 signifies the third section in the second chapter. Extensive sections are usually divided into subsections identified by upper-case common letters A, B, C, etc. Equations, definitions, theorems, corollaries, lemmas, examples, exercises, figures, and special remarks are assigned monotonically increasing numerals which identify the chapter, section, and item number. For example, Theorem 4.4.7 denotes the seventh identified item in the fourth section of Chapter 4. This theorem is followed by Eq. (4.4.8), the eighth identified item in the same section. Within a given chapter, figures are identified by upper-case letters A, B, C, etc., while outside of the chapter, the same figure is identified by the above numbering scheme. Finally, the end of a proof or of an example is signified by the symbol ■.

Suggested Course Outlines

Because of the flexibility described above, this book can be used either in a one-semester course, or a two-semester course. In either case, mastery of the material presented will give the student an appreciation of the power and the beauty of the axiomatic method; will increase the student's ability to construct proofs; will enable the student to distinguish between purely algebraic and topological structures and combinations of such structures in mathematical systems; and of course, it will broaden the student's background in algebra and analysis.

A one-semester course

Chapters 1, 3, 4, 5, and Sections 6.1 and 6.11 in Chapter 6 can serve as the basis for a one-semester course, emphasizing basic aspects of *Linear Algebra* and *Analysis* in a metric space setting.

The coverage of *Chapter 1* should concentrate primarily on functions (Section 1.2) and relations and equivalence relations (Section 1.3), while the material concerning sets (Section 1.1) and operations on sets (Section 1.4) may be covered as reading assignments. On the other hand, Section 1.5 (on mathematical systems) merits formal coverage, since it gives the student a good overview of the book's aims and contents.

The material in this book has been organized so that *Chapter 2*, which addresses the important algebraic structures encountered in *Abstract Algebra*, may be omitted without any loss of continuity. In a one-semester course emphasizing Linear Algebra, this chapter may be omitted in its entirety.

In *Chapter 3*, which addresses general vector spaces and linear transformations, the material concerning linear spaces (Section 3.1), linear subspaces and direct sums (Section 3.2), linear independence and bases (Section 3.3), and linear transformations (Section 3.4) should be covered in its entirety, while selected topics on linear functionals (Section 3.5), bilinear functionals (Section 3.6), and projections (Section 3.7) should be deferred until they are required in Chapter 4.

Chapter 4 addresses finite-dimensional vector spaces and linear transformations (matrices) defined on such spaces. The material on determinants (Section 4.4) and some of the material concerning linear transformations on Euclidean vector spaces (Subsections 4.10D and 4.10E), as well as applications to ordinary differential equations (Section 4.11) may be omitted without any loss of continuity. The emphasis in this chapter should be on coordinate representations of vectors (Section 4.1), the representation of linear transformations by matrices and the properties of matrices (Section 4.2), equivalence and similarity of matrices (Section 4.3), eigenvalues and eigenvectors (Section 4.5), some canonical forms of matrices (Section 4.6), minimal polynomials, nilpotent operators and the Jordan canonical form (Section 4.7), bilinear functionals and congruence (Section 4.8), Euclidean vector spaces (Section 4.9), and linear transformations on Euclidean vector spaces (Subsections 4.10A, 4.10B, and 4.10C).

Chapter 5 addresses metric spaces, which constitute some of the most important topological spaces. In a one-semester course, the emphasis in this chapter should be on the definition of metric space and the presentation of important classes of metric spaces (Sections 5.1 and 5.3), open and closed sets (Section 5.4), complete metric spaces (Section 5.5), compactness (Section 5.6), and continuous functions (Section 5.7). The development of many classes of metric spaces requires important inequalities, including the Hölder and the Minkowski inequalities for finite and infinite sums and for integrals. These are presented in Section 5.2 and need to be included in the course. Sections 5.8 and 5.10 address specific applications and may be omitted without any loss of continuity. However, time permitting, the material in Section 5.9, concerning equivalent and homeomorphic metric spaces and topological spaces, should be considered for inclusion in the course, since it provides the student a glimpse into other areas of mathematics.

To demonstrate mathematical systems endowed with both algebraic and topological structures, the one-semester course should include the material of Sections 6.1 and 6.2 in *Chapter 6*, concerning normed linear spaces (resp., Banach spaces) and inner product spaces (resp., Hilbert spaces), respectively.

A two-semester course

In addition to the material outlined above for a one-semester course, a two-semester course should include most of the material in Chapters 2, 6, and 7.

Chapter 2 addresses algebraic structures. The coverage of semigroups and groups, rings and fields, and modules, vector spaces and algebras (Section 2.1) should be in sufficient detail to give the student an appreciation of the various algebraic structures summarized in Figure B on page 61. Important mappings defined on these algebraic structures (homomorphisms) should also be emphasized (Section 2.2) in a two-semester course, as should the brief treatment of polynomials in Section 2.3.

The first ten sections of *Chapter 6* address normed linear spaces (resp., Banach spaces) while the next four sections address inner product spaces (resp., Hilbert spaces). The last section of this chapter, which includes applications (to random variables and estimates of random variables), may be omitted without any loss of continuity. The material concerning normed linear spaces (Section 6.1), linear subspaces (Section 6.2), infinite series (Section 6.3), convex sets (Section 6.4), linear functionals (Section 6.5), finite-dimensional spaces (Section 6.6), inner product spaces (Section 6.11), orthogonal complements (Section 6.12), and Fourier series (Section 6.13) should be covered in its entirety. Coverage of the material on geometric aspects of linear functionals (Section 6.7), extensions of linear functionals (Section 6.8), dual space and second dual space (Section 6.9), weak convergence (Section 6.10), and the Riesz representation theorem (Section 6.14) should be selective and tailored to the availability of time and the students' areas of interest. (For example, students interested in optimization and estimation problems may want a detailed coverage of the Hahn–Banach theorem included in Section 6.8.)

Chapter 7 addresses (bounded) linear operators defined on Banach and Hilbert spaces. The first nine sections of this chapter should be covered in their entirety in a two-semester course. The material of this chapter includes bounded linear transformations (Section 7.1), inverses (Section 7.2), conjugate and adjoint operators (Section 7.3), Hermitian operators (Section 7.4), normal, projection, unitary and isometric operators (Section 7.5), the spectrum of an operator (Section 7.6), completely continuous operators (Section 7.7), the spectral theorem for completely continuous normal operators (Section 7.8), and differentiation of (not necessarily linear and bounded) operators (Section 7.9). The last section, which includes applications to integral equations, an example from optimal control, and minimization of functionals by the method of steepest descent, may be omitted without loss of continuity.

Both one-semester and two-semester courses offered by the present authors, based on this book, usually included a project conducted by each course participant to demonstrate the applicability of the course material. Each project involved a formal presentation to the entire class at the end of the semester.

The courses described above were also offered using the Socratic method, following the outlines given above. These courses typically involved half a dozen participants. While most of the material was self taught by the students themselves, the classroom meetings served as a forum for guidance, clarifications, and challenges by the teacher, usually resulting in lively discussions of the subject on hand not only among teacher and students, but also among students themselves.

For the current printing of this book, we have created a supplementary website of additional resources for students and instructors: http://Michel.Herget.net. Available at this website are additional current references concerning the subject matter of the book and a list of several areas of applications (including references). Since the latter reflects mostly the authors' interests, it is by definition rather subjective. Among several additional items, the website also includes some reviews of the present book. In this regard, the authors would like to invite readers to submit reviews of their own for inclusion into the website.

The present publication of *Algebra and Analysis for Engineers and Scientists* was made possible primarily because of Tom Grasso, Birkhäuser's Computational Sciences and Engineering Editor, whom we would like to thank for his considerations and professionalism.

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