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Texts in Applied Mathematics

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Introduction to the Foundations of Applied Mathematics

Springer

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To Colette, Matthew and Marianna

Preface

FOAM. This acronym has been used for over fifty years at Rensselaer to designate an upper-division course entitled, Foundations of Applied Mathematics. This course was started by George Handelman in 1956, when he came to Rensselaer from the Carnegie Institute of Technology. His objective was to closely integrate mathematical and physical reasoning, and in the process enable students to obtain a qualitative understanding of the world we live in. FOAM was soon taken over by a young faculty member, Lee Segel. About this time a similar course, Introduction to Applied Mathematics, was introduced by Chia-Ch'iao Lin at the Massachusetts Institute of Technology. Together Lin and Segel, with help from Handelman, produced one of the landmark textbooks in applied mathematics, *Mathematics Applied to Deterministic Problems in the Natural Sciences.* This was originally published in 1974, and republished in 1988 by the Society for Industrial and Applied Mathematics, in their Classics Series.

This textbook comes from the author teaching FOAM over the last few years. In this sense, it is an updated version of the Lin and Segel textbook. The objective is definitely the same, which is the construction, analysis, and interpretation of mathematical models to help us understand the world we live in. However, there are some significant differences. Lin and Segel, like many recent modeling books, is based on a case study format. This means that the mathematical ideas are introduced in the context of a particular application. There are certainly good reasons why this is done, and one is the immediate relevance of the mathematics. There are also disadvantages, and one pointed out by Lin and Segel is the fragmentary nature of the development. However, there is another, more important reason for not following a case studies approach. Science evolves, and this means that the problems of current interest continually change. What does not change as quickly is the approach used to derive the relevant mathematical models, and the methods used to analyze the models. Consequently, this book is written in such a way as to establish the mathematical ideas underlying model development independently of a specific application. This does not mean applications are not considered, they are, and connections with experiment are a staple of this book.

The first two chapters establish some of the basic mathematical tools that are needed. The model development starts in Chapter 3, with the study of kinetics. The goal of this chapter is to understand how to model interacting populations. This does not account for the spatial motion of the populations, and this is the objective of Chapters 4 and 5. What remains is to account for the forces in the system, and this is done in Chapter 6. The last three chapters concern the application to specific problems and the generalization of the material to more geometrically realistic systems. The book, as well as the individual chapters, is written in such a way that the material becomes more sophisticated as you progress. This provides some flexibility in how the book is used, allowing consideration for the breadth and depth of the material covered.

The principal objective of this book is the derivation and analysis of mathematical models. Consequently, after deriving a model, it is necessary to have a way to solve the resulting mathematical problem. A few of the methods developed here are standard topics in upper-division applied math courses, and in this sense there is some overlap with the material covered in those courses. Examples are the Fourier and Laplace transforms, and the method of characteristics. On the other hand, other methods that are used here are not standard, and this includes perturbation approximations and similarity solutions. There are also unique methods, not found in traditional textbooks, that rely on both the mathematical and physical characteristics of the problem.

The prerequisite for this text is a lower-division course in differential equations. The implication is that you have also taken two or three semesters of calculus, which includes some component of matrix algebra. The one topic from calculus that is absolutely essential is Taylor's theorem, and for this reason a short summary is included in the appendix. Some of the more sophisticated results from calculus, related to multidimensional integral theorems, are not needed until Chapter 8.

To learn mathematics you must work out problems, and for this reason the exercises in the text are important. They vary in their difficulty, and cover most of the topics in the chapter. Some of the answers are available, and can be found at www.holmes.rpi.edu. This web page also contains a typos list.

I would like to express my gratitude to the many students who have taken my FOAM course at Rensselaer. They helped me immeasurably in understanding the subject, and provided much-needed encouragement to write this book. It is also a pleasure to acknowledge the suggestions of John Ringland, and his students, who read an early version of the manuscript.

Troy, New York *Mark H. Holmes*
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