S*pringer* **M***onographs* in **M***athematics*

For further volumes:
www.springer.com/series/3733

Joachim Hilgert · Karl-Hermann Neeb

Structure and Geometry of Lie Groups

Springer

Joachim Hilgert
Mathematics Institute
University of Paderborn
Warburgerstr. 100
33095 Paderborn
Germany
hilgert@math.uni-paderborn.de

Karl-Hermann Neeb
Department of Mathematics
Friedrich-Alexander Universität
Erlangen-Nürnberg
Cauerstrasse 11
91054 Erlangen
Germany
neeb@mi.uni-erlangen.de

ISSN 1439-7382 Springer Monographs in Mathematics
ISBN 978-0-387-84793-1 e-ISBN 978-0-387-84794-8
DOI 10.1007/978-0-387-84794-8
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2011942060

Mathematics Subject Classification (2010): 7Bxx, 22Exx, 22Fxx

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Nowadays there are plenty of textbooks on Lie groups to choose from, so we feel we should explain why we decided to add another one to the row. Most of the readily available books on Lie groups either aim at an elementary introduction mostly restricted to matrix groups, or else they try to provide the background on semisimple Lie groups needed in harmonic analysis and unitary representation theory with as little general theory as possible. In [HN91], we tried to exhibit the basic principles of Lie theory rather than specific material, stressing the exponential function as the means of translating problems and solutions between the global and the infinitesimal level. In that book, written in German for German students who typically do not know differential geometry but are well versed in advanced linear algebra, we avoided abstract differentiable manifolds by combining matrix groups with covering arguments. Having introduced the basic principles, we demonstrated their power by proving a number of standard and not so standard results on the structure of Lie groups. The choice of results included owed a lot to Hochschild's book [Ho65], which even then was not so easy to come by.

This book builds on [HN91], but after twenty years of teaching and research in Lie theory we found it indispensable to also have the differential geometry of Lie groups available. Even though this is not apparent from the text, the reason for this is the large number of applications and further developments of Lie theory in which differential manifolds are essential. Moreover, we decided to include a number of structural results we found to be useful in the past but not readily available in the textbook literature. The basic line of thought now is:

- Simple examples: Matrix groups
- Tools from algebra: Lie algebras
- Tools from geometry: Smooth manifolds
- The basic principles: Lie groups, their Lie algebras, and the exponential function
- Structure theory: General Lie groups and special classes
- Testing methods on examples: The topology of classical groups
- A slight extension: Several connected components

While this book offers plenty of tested material for various introductory courses such as *Matrix Groups*, *Lie Groups*, *Lie Algebras*, or *Differentiable*

Manifolds, it is not a textbook to follow from A to Z (see page 6 for teaching suggestions). Moreover, it contains advanced material one would not typically include in a first course. In fact, some of the advanced material has not appeared in any monograph before. This and the fact that we wanted the book to be self contained is the reason for its considerable length. In order to still keep the work within reasonable limits, for some topics which are well covered in the textbook literature, we decided to include only what was needed for the further developments in the book. This applies, e.g., to the standard structure and classification theories of semisimple Lie algebras. Thus we do *not* want to suggest that this book can replace previous textbooks. It is meant rather to be a *true addition* to the existing textbook literature on Lie groups.

As was mentioned before, we are well aware of the fact that modern mathematics abounds with applications of Lie theory while this book hardly mentions any of them. The reason is that most applications require additional knowledge of the field in which these applications occur, so describing them would have meant either extensive storytelling or else a considerable expansion in length of this book. Neither option seemed attractive to us, so we leave it to future books to give detailed accounts of the beautiful ways in which Lie theory enters different fields of mathematics.

Even though there was a forerunner book and many lecture notes produced for various courses over the years, in compiling this text we produced many typos and made some mistakes. Many of those were shown to us by a small army of enthusiastic proofreaders to whom we are extremely grateful: Hanno Becker, Jan Emonds, Hasan Gündogan, Michael Klotz, Stéphane Merigon, Norman Metzner, Wolfgang Palzer, Matthias Peter, Niklas Schaeffer, Henrik Seppänen, and Stefan Wagner read major parts of the manuscript, and there were others who looked at particular sections. Of course, we know that the final version of the book will also contain mistakes, and we assume full responsibility for those.

We also would like to thank Ilka Agricola and Thomas Friedrich for some background information on the early history of Lie theory.

Paderborn *Joachim Hilgert*
Erlangen *Karl-Hermann Neeb*

Table of Contents

Part V Appendices