Universitext

For other titles in this series, go to
http://www.springer.com/series/223

J.M. Aarts

Plane and Solid Geometry

Springer

J.M. Aarts
Delft University of Technology
Mediamatics
The Netherlands
j.a.aarts@ewi.tudelft.nl

Translator:
Reinie Erné
Leiden, The Netherlands
erne@math.leidenuniv.nl

ISBN: 978-0-387-78240-9 e-ISBN: 978-0-387-78241-6
DOI: 10.1007/978-0-387-78241-6

Library of Congress Control Number: 2008935537

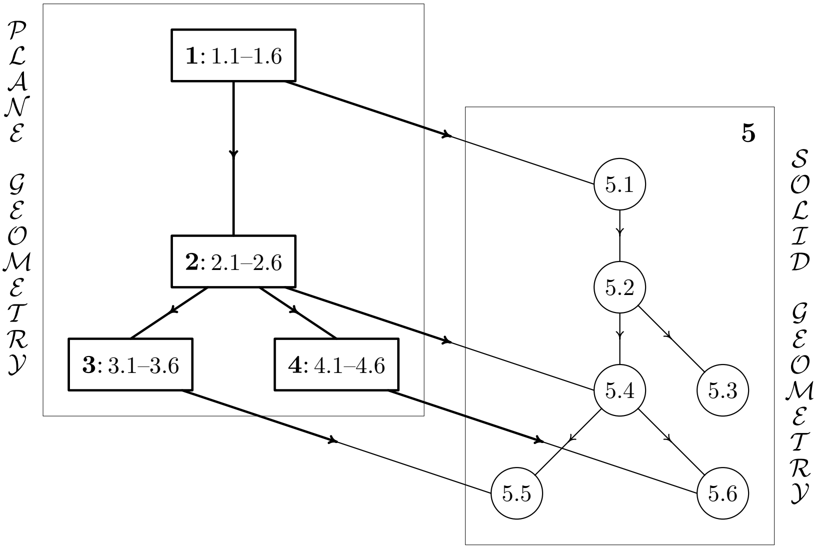
Mathematics Subject Classification (2000): 51-xx

This is a translation of the Dutch, *Meetkunde*, originally published by Epsilon–Uitgaven, 2000.

Printed on acid-free paper

springer.com

To Robert J. and Lucie P.


PLANE GEOMETRY
1: 1.1–1.6
2: 2.1–2.6
3: 3.1–3.6
4: 4.1–4.6
5
5.1
5.2
5.3
5.4
5.5
5.6
SOLID GEOMETRY

Preface

Nature and the world around us that we ourselves design, furnish, and build contain many geometric patterns and structures. This is one of the reasons that geometry should be studied at school. At first, the study of geometry is experimental. Results are taught and used in numerous examples. Only later do proofs come into play. But are these proofs truly necessary, or can we do without them? A natural answer is that every statement must be provided with a proof, because we want to know whether it is true. However, it is clear that the less experienced student may become frustrated by the presence of too many proofs. Only later will the student understand that proofs not only show the correctness of a statement, but also provide better insight into the relations among various properties of the objects that are being studied. Learning statements without proofs, you risk not being able to see the forest for the trees. For this reason, we will pay much attention to a careful presentation of proofs in this book. In the development of the theory of plane geometry there are, however, many tricky questions, especially at the beginning. The presentation of proofs at that stage is in general more concealing than revealing.

My first objective in writing this book has been to give an accessible exposition of the most common notions and properties of elementary Euclidean geometry in dimensions two and three. These include, in particular, special lines in triangles, congruence criteria, transformations, circles, and conics. All this can be found in the first hundred pages, Chapters 1 and 2. I also briefly discuss fractals and Voronoi diagrams, and give a detailed account of symmetry, cycloids, and notions of solid geometry. Chapters 1, 2, and 4 present a survey of the results of plane geometry at an intermediate level. Chapters 3 and 5 present more advanced topics. The first four chapters deal solely with plane geometry, while the fifth, and final, chapter discusses solid geometry.

Geometry is a useful subject with many applications. But what makes the study of geometry so captivating is the feeling of wonder that comes over you when you ponder questions such as, “Why are those three special lines concurrent?” and “Why do so many special points lie on the same circle?” the feeling that the world is truly a beautiful place! To mention just one example, the nine-point circle contains nine special points of a triangle (Example 2.46), touches both the incircle and the three excircles of the triangle (Theorem 4.30), and is the incircle of the deltoid that is the envelope of the Simson lines of the triangle (Theorem 4.58). In choosing the topics for this book I used the following criteria: is a topic useful, it is surprising, or is it in vogue.

When I started writing this book I aimed to present a complete account of Euclidean geometry in dimensions two and three. But soon I discovered that I could not discuss the foundations of geometry without losing momentum. So I decided to start the discussion at an intermediate stage and to make a special choice of basic assumptions on which to build the theory. Let me briefly mention these assumptions. The first basic assumption is that the plane admits a distance function that assigns a distance to any two points. Using this distance function we can define straight lines. We then make the basic assumption that every straight line is an isometric copy of the real line. On the basis of this assumption we can use various properties of the real numbers to obtain rather simple proofs of results whose traditional proofs are often quite complicated. The next basic assumption is about the existence of parallel lines. Another basic assumption concerns the existence of perpendicular lines; whether two given lines are perpendicular is decided with the help of the inverse Pythagorean theorem. The last basic assumption concerns the measurement of angles. Starting with these basic assumptions, we develop plane geometry. The introduction of local coordinates becomes relatively easy. Needless to say, the use of coordinates simplifies many proofs. Our presentation of the results in this book is not always strictly sequential. Several properties of the nine-point circle, for example, are discussed before the definitions of the circle and circumcircle are given.

With the approach I just mentioned, we can introduce the basic notions and elementary properties of figures in plane geometry in a way that is brief and to the point. We do this in Chapter 1, which also contains a survey of the properties required to read the subsequent chapters.

In Chapter 2, we study distance-preserving maps of the plane, also called isometries. We show that every isometry is a reflection in a line, a translation, a rotation, or a glide reflection. We also introduce the notion of congruence, and give the classic congruence criteria. Next, we give a detailed exposition of the notion of orientation. Finally, we discuss similarities and their role in the theory of fractals.

The subsequent chapters of the book can be read independently of each other. The diagram on page ii shows the relations between various parts of the book. Chapter 3 starts with the study of the symmetry groups of a number of simple plane figures. The main object of the chapter is the study of frieze patterns and periodic tilings, which we classify according to their symmetry groups. To analyze these, we use Voronoi diagrams, which also provide an elegant way of introducing conic sections.

In Chapter 4, we study various curves, namely the circle, conic sections, and cycloids. In the first two sections we give a detailed account of the most important properties of the circle and of the trigonometric functions. This is followed by a discussion of some unexpected applications of the inversion of the plane in a circle. We then turn to the conic sections. In order to obtain a transparent derivation of the equation of a tangent to a conic, we introduce the notion of polarity, for which we need homogeneous coordinates. The last section of the chapter concerns cycloids. We explain the relation between three seemingly unrelated topics: the nine-point circle, the Simson line, and the deltoid.

Chapter 5 presents a bird's-eye view of several topics of solid geometry. We first briefly cover the subjects of the previous chapters, adapted to solid geometry. One of the sections is devoted to a discussion of several ways to make drawings of solid figures. Using reflections in a plane, we can list the types of isometries of Euclidean 3-space: reflections in a plane, translations, rotations, glide reflections, reflections in a point, improper rotations, and screws. In our review of the symmetry of solid figures, we discuss all regular and two semiregular polyhedra. This discussion results in a list of all finite subgroups of the group of direct isometries of 3-space. In the final section of the chapter, we study quadrics. In particular, we consider straight lines on quadrics, the relation between quadrics and conic sections, and the classification of quadrics. The Hessian, which is used in calculus to study stationary points, is mentioned as an application of the latter. We conclude the book with an appendix listing the basic assumptions for both plane and solid geometry.

Math books are not like novels. Reading them becomes fun only when you take a pen and paper and work out the results yourself. For this reason I have included some two hundred exercises in this book. It was not my intention to include brainteasers, but some time and effort will undoubtedly be required to solve the problems. There is no need to become frustrated by this, for devoting time and effort to problemsolving is rewarding. The exercises are related to the material of the section they follow. The solution of an exercise is never tricky, but mostly rather straightforward. Most exercises are just statements, without phrases such as *prove this* or *show that*. The reader is invited to provide a proof of the statement. If an exercise turns out to be too difficult, it should be skipped and reexamined at a later moment. A similar remark applies to the proofs that are presented in this book. Quite often, they look much harder at first reading than they really are. Frequently in mathematics, a problem or proof that was obscure yesterday can be grasped today, and will be trivial tomorrow.

In writing this book, I used the notes from several courses and talks I have given in Delft and in Amsterdam. Parts of Chapters 2 and 3 were developed for a course given to architecture students, and parts of Chapters 4 and 5 were for a course for mechanical-engineering students, both in Delft. Certain subjects, in particular Voronoi diagrams, conics, quadrics, and fractals, were used in courses for secondary-school teachers.

I received help from different directions. Agnes Verweij was coauthor of the material for the course for mechanical-engineering students. She also read through my contributions to the courses for secondary-school teachers; her comments have always been very helpful. K.P. Hart stood by to help on any LaTeX problems I might encounter. He also taught me to work with `mfpic`, the program developed by Tom Leathrum, which I used to make the figures for this book. Eva Coplakova conscientiously worked her way through the whole manuscript, including all the problems; her remarks were very useful. I would like to thank as well everyone else who has helped me.

November 2007 *Jan Aarts*
Delft

Contents