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# INTRODUCTION TO APPLIED OPTIMIZATION

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## *Aims and Scope*

Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.

The series *Springer Optimization and Its Applications* publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multi-objective programming, description of software packages, approximation techniques and heuristic approaches.

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# INTRODUCTION TO APPLIED OPTIMIZATION

Second Edition

By

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 Springer

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*To my parents Leela and Murlidhar Diwekar for teaching me  
to be optimistic and to dream.*

*To my husband Sanjay Joag for supporting my dreams and  
making them a reality.*

*And*

*To my niece Ananya whose innocence and charm provide  
optimism for the future.*

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## FOREWORD

Optimization has pervaded all spheres of human endeavor. Although optimization has been practiced in some form or other from the early prehistoric era, this area has seen progressive growth during the last five decades. Modern society lives not only in an environment of intense competition but is also constrained to plan its growth in a sustainable manner with due concern for conservation of resources. Thus, it has become imperative to plan, design, operate, and manage resources and assets in an optimal manner. Early approaches have been to optimize individual activities in a standalone manner, however, the current trend is towards an integrated approach: integrating synthesis and design, design and control, production planning, scheduling, and control. The functioning of a system may be governed by multiple performance objectives. Optimization of such systems will call for special strategies for handling the multiple objectives to provide solutions closer to the systems requirement. Uncertainty and variability are two issues which render optimal decision making difficult. Optimization under uncertainty would become increasingly important if one is to get the best out of a system plagued by uncertain components. These issues have thrown up a large number of challenging optimization problems which need to be resolved with a set of existing and newly evolving optimization tools.

Optimization theory had evolved initially to provide generic solutions to optimization problems in linear, nonlinear, unconstrained, and constrained domains. These optimization problems were often called mathematical programming problems with two distinctive classifications, namely linear and nonlinear programming problems. Although the early generation of programming problems were based on continuous variables, various classes of assignment and design problems required handling of both integer and continuous variables leading to mixed integer linear and nonlinear programming problems (MILP and MINLP). The quest to seek global optima has prompted researchers to develop new optimization approaches which do not get stuck at a local optimum, a failing of many of the mathematical programming methods. Genetic algorithms derived from biology and simulated annealing inspired by optimality

of the annealing process are two such potent methods which have emerged in recent years. The developments in computing technology have placed at the disposal of the user a wide array of optimization codes with varying degrees of rigor and sophistication. The challenges to the user are manifold. How to set up an optimization problem? What is the most suitable optimization method to use? How to perform sensitivity analysis? An intrepid user may also want to extend the capabilities of an existing optimization method or integrate the features of two or more optimization methods to come up with more efficient optimization methodologies.

This book, appropriately titled *Introduction to Applied Optimization*, has addressed all the issues stated above in an elegant manner. The book has been structured to cover all key areas of optimization namely deterministic and stochastic optimization, and single and multiobjective optimization. In keeping with the application focus of the book, the reader is provided with deep insights into key aspects of an optimization problem: problem formulation, basic principles and structure of various optimization techniques, and computational aspects.

The book begins with a historical perspective on the evolution of optimization followed by identification of key components of an optimization problem and its mathematical formulation. Types of optimization problems that can occur and the software codes available to solve these problems are presented. The book then moves on to treat in the next two chapters two major optimization methods, namely linear programming and nonlinear programming. Simple introductory examples are used to illustrate graphically the characteristics of the feasible region and location of optima. The simplex method used for the solution of the LP problem is described in great detail. The author has used an innovative example to develop the Karush–Kuhn–Tucker conditions for NLP problems. Lagrangian formulation has been used to develop the relationships between primal–dual problems. The transition from the continuous to discrete optimization problem is made in Chapter 4. The distinctive character of the solution to the discrete optimization problem is demonstrated graphically with a suitable example. The efficacy of the branch-and-bound method for solution of MILP and MINLP problems is brought out very clearly. Decomposition methods based on generalized Bender’s decomposition (GBD) and outer approximation (OA) are projected as efficient approaches for solution of MILP and MINLP problems. Developing optimal solutions using simulated annealing and genetic algorithms are also explained in great detail. The potential of combining simulated annealing and nonlinear programming (SA-NLP) to generate more efficient solutions for MINLP problems is stressed with suitable examples. Chapter 5 deals with strategies for optimization under uncertainty. The strategy of using the mean value of a random variable for optimization is shown to be suboptimal. Using probabilistic information on the uncertain variable, various measures such as value of stochastic solution (VSS) and expected value of perfect information (EVPI) are developed. The optimization problem with recourse is analyzed. Two policies are considered, namely “here and

now” and “wait and see”. The development of chance constrained programming and L-shaped decomposition methods using probability information is shown. For simplification of optimization under uncertainty, use of sampling techniques for scanning the uncertain parameter space is advocated. Among the various sampling methods analyzed, the Hammersley sequence sampling is shown to be the most efficient. The stochastic annealing algorithm with adaptive choice of sample size is shown as an efficient method for handling stochastic optimization problems.

Multiobjective optimization is treated in the next chapter. The process of identification of a nondominated set from the set of feasible solutions is presented. Three methods, namely the weighting method, constraint method and goal programming method are discussed. STA-NLP framework is proposed as an alternate approach to handle multiobjective optimization problems.

The book ends with a treatment of optimal control in Chapter 7. The first part deals with well-known methods such as the calculus of variations, maximum principle, and dynamic programming. The next part deals with stochastic dynamic optimization. Stochastic formulation of dynamic programming is done using Ito’s lemma. The book concludes with a detailed study of the dynamic optimization of batch distillation. The thorough treatment of the stochastic distillation case should provide a revealing study for the reader interested in solving dynamic optimization problems under uncertainty.

The material in the book has been carefully prepared to keep the theoretical development to a minimal level while focusing on the principles and implementation aspects of various algorithms. Numerous examples have been given to lend clarity to the presentations. Dr. Diwekar’s own vast research experience in nonlinear optimization, optimization under uncertainty, process synthesis, and dynamic optimization has helped in focusing the reader’s attention to critical issues associated with various classes of optimization problems. She has used the hazardous waste blending problem on which she has done considerable research as a complex enough process for testing the efficacy of various optimization methods. This example is used very skillfully to demonstrate the strengths and weaknesses of various optimization methods.

The book with its wide coverage of most of the well-established and emerging optimization methods will be a valuable addition to the optimization literature. The book will be a valuable guide and reference material to a wide cross-section of the user community comprising students, faculty, researchers, practitioners, designers, and planners.

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Department of Chemical Engineering  
Indian Institute of Technology  
Bombay, India  
20 November, 2002

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## PREFACE TO THE SECOND EDITION

I am happy to present the second edition of this book. In this second edition, I have updated all the chapters and additional material has been added in Chapter 3 and Chapter 7. New examples have also been added in various chapters. The solution manual and case studies for this book are available online on the Springer website with the book link <http://www.springer.com/math/book/978-0-387-76634-8>.

This book would not have been possible without the constant support from my husband Dr. Sanjay Joag, and my sisters Dr. Anjali Diwekar and Dr. Prajakta Sambarey. Thanks are due to my graduate students Francesco Baratto, Saadet Ulas, Karthik Subramanyan, Weiyu Xu, and Yogendra Shastri for providing feedback on the first edition. Thanks are also due to the many readers around the world who sent valuable feedback.

*Urmila M. Diwekar*  
Clarendon Hills, Illinois  
February, 2007

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आशा नाम मनुष्याणां काचिदाश्चर्यं शृङ्खला ।

यया बध्दा प्रधावन्ति मुक्तां तिष्ठन्ति पङ्गवत् ॥

—Sanskrit Subhashita

*What an amazing chain the word hope is for humans, tied by it they surge forward, and unbound from it, they sit like the lame.*

This book was written during some difficult times. My parents have taught me to be hopeful and optimistic. That and the constant support from my husband, Dr. Sanjay Joag, kept me going. This book would not exist without the personal support and encouragement I received from him, my parents, and my mother-in-law, Leela Joag.

It meant a lot to me that this book received a foreword from my advisor and guru, Professor K. P. Madhavan. He not only taught me the first course on optimization but also how to be a disciplined and rigorous researcher. My heartfelt thanks to him.

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To my parents for their untiring love and their lifetime of hard work behind the greatest gift to me—education—to my husband for his constant love, encouragement, and his unconditional sacrifices to support my career, and to my innocent and charming niece Ananya for being what she is, I dedicate this work.

*Urmila M. Diwekar  
Chicago, Illinois  
December, 2002*

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