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Wavelet Methods in Statistics with R

 Springer

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To Philippa, Lucy, Suzannah, Mum and Dad.

Preface

When Zhou Enlai, Premier of the People's Republic of China (1949–1976), was asked his opinion of the French Revolution (1789–1799) he replied “It's too early to tell”, see Rosenberg (1999). I believe that the same can be said about wavelets. Although particular wavelets were discovered many years ago, the substantial body of literature that we might today call ‘wavelet theory’ began to be established during the 1980s. Wavelets were introduced into statistics during the late 1980s and early 1990s, and they were initially popular in the curve estimation literature. From there they spread in different ways to many areas such as survival analysis, statistical time series analysis, statistical image processing, inverse problems, and variance stabilization.

The French Revolution was also the historical backdrop for the introduction of Fourier series which itself raised considerable objections from the scientific establishment of the day, see Westheimer (2001). Despite those early objections, we find that, 200 years later, many new Fourier techniques are regularly being invented in many different fields. Wavelets are also a true scientific revolution. Some of their interesting features are easy to appreciate: e.g., multiscale, localization, or speed. Other important aspects, such as the unconditional basis property, deserve to be better known. I hope that this book, in some small way, enables the creation of many new wavelet methods. Wavelet methods will be developed and important for another 200 years!

This book is about the role of wavelet methods in statistics. My aim is to cover the main areas in statistics where wavelets have found a use or have potential. Another aim is the promotion of the *use* of wavelet methods as well as their *description*. Hence, the book is centred around the freeware **R** and **WaveThresh** software packages, which will enable readers to learn about statistical wavelet methods, *use* them, and *modify* them for their own use. Hence, this book is like a traditional monograph in that it attempts to cover a wide range of techniques, but, necessarily, the coverage is biased towards areas that I and **WaveThresh** have been involved in. A feature is that the code for nearly all the figures in this book is available from the **WaveThresh**

website. Hence, I hope that this book (at least) partially meets the criteria of ‘reproducible research’ as promoted by Buckheit and Donoho (1995).

Most of `WaveThresh` was written by me. However, many people contributed significant amounts of code and have generously agreed for this to be distributed within `WaveThresh`. I would like to thank Felix Abramovich (FDR thresholding), Stuart Barber (complex-valued wavelets and thresholding, Bayesian wavelet credible interval), Tim Downie (multiple wavelets), Idris Eckley (2D locally stationary wavelet processes), Piotr Fryzlewicz (Haar–Fisz transform for Poisson), Arne Kovac (wavelet shrinkage for irregular data), Todd Ogden (change-point thresholding), Theofanis Sapatinas (Donoho and Johnstone test functions, some wavelet packet time series code, `BayesThresh` thresholding), Bernard Silverman (real FFT), David Herrick (wavelet density estimation), and Brani Vidakovic (Daubechies-Lagarias algorithm). Many other people have written add-ons, improvements, and extensions, and these are mentioned in the text where they occur. I would like to thank Anthony Davison for supplying his group’s `SBand` code.

I am grateful to A. Black and D. Moshal of the Dept. of Anaesthesia, Bristol University for supplying the plethysmography data, to P. Fleming, A. Sawczenko, and J. Young of the Bristol Institute of Child Health for supplying the infant ECG/sleep state data, to the Montserrat Volcano Observatory and Willy Aspinall, of Aspinall and Associates, for the `RSAM` data.

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