

Surveys and Tutorials in the Applied Mathematical Sciences

Volume 3

Series Editors

S.S. Antman, J.E. Marsden, L. Sirovich

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Surveys and Tutorials in the Applied Mathematical Sciences

Volume 3

Series Editors

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Thomas Erneux

Applied Delay Differential Equations

 Springer

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Preface

Qu'est-ce que le passé, sinon du présent qui est en retard?
Pierre Dac (l'os à moelle, March 1940)

Modeling automatic engines or physiological systems often involves the idea of control because feedback is used in order to maintain a stable state. But much of this feedback require a finite time to sense information and react to it. A popular way to describe this process is to formulate a delay differential equation (DDE) where the evolution of a dependent variable at time t depends on its value at time $t - \tau$. Unfortunately, solving a DDE is a mathematically difficult task. Over the past decade, rapid advances in computational power have revived interest in DDEs. Previously known equations are investigated allowing a better physical understanding of old problems. In addition, new areas of research have appeared. This is, for example, the case of lasers subject to optical feedback, the delayed control of container cranes, or the real-time synthesis of musical instruments.

Oscillatory instabilities are frequently associated with systems described by DDEs. The motivation to study these oscillations then depends on the background of the researcher. For some, these oscillatory instabilities are viewed as a limitation to the performance of a particular device that must be avoided or possibly controlled. In contrast, other researchers have put the unstable behavior to good use making practical devices such as high-frequency optical oscillators.

New mathematical tools and reliable computer software techniques have been developed for DDEs. Here, preference is given to analytical approaches known collectively as asymptotic methods [22, 124], the most

useful techniques for finding approximate solutions to equations. It is a revised and largely expanded version of a series of lectures first given at the Université Libre de Bruxelles in 2002–2003, at the Université Joseph Fourier (Grenoble) in 2003, and, more recently, at the University of Utah in 2007. The minimum prerequisites for this book are a facility with calculus, experience with differential equations, and an elementary knowledge of bifurcation theory. The unusual format of this textbook, avoiding rigorous mathematical proofs and concentrating on applications, aims to introduce beginning students as well as experienced researchers to the large variety of phenomena described by DDEs. It has no ambition to review the rich field of DDEs and references have been selected for their historical impact or for the experiments they are describing.

One novelty in this volume is the place given to the figures. They help in understanding the scientific background of a specific application and how a mathematical model is derived. In addition, computer plots compare exact and approximative solutions illustrating the efficiency of the analytical method. The mathematical computations are described in as friendly a manner as possible.

DDE models are used by biologists, physicists, and engineers with different objectives and expectations. This text is meant to serve as an introduction to the rich variety of applications and could be used in a modeling course on DDEs. Some selected parts of this book could provide material for a class on singular perturbation techniques or to stimulate a differential equation class. It is my experience that the combination of illustrations using a projector and compact computations on the blackboard works well to attract the attention of the audience.

There are many colleagues to thank for their interest, suggestions, and contributions to this book. I first wish to thank Tamás Kalmár-Nagy and John Milton who gave me precious details on comparisons between experiments and theory. The collaboration with Dirk Roose and his group during the years 2000–2003 was a successful experience combining new analytical and numerical approaches. Applied mathematicians Don Cohen, Michael Mackey, and John Ockendon strongly encouraged me to go forward with this project. The lectures by Gabor Stépan on mechanical engineering problems, by Yang Kuang and Stephen Gourley on population models, and by John Mallet-Paret, Roger Nussbaum, and Hans-Otto Walther on state-dependent delay equations had a strong impact on me. Of course, I am deeply indebted to my friends in the laser community who in 1993 introduced me to the world of optical feedback. Tom Gavrielides, Vassilios Kovanis, and Daan Lenstra educated me on the complexities of the Lang and Kobayashi equations and Ingo Fisher, Eric Lacot, Laurent Larger, Raj Roy, and David Sukow patiently explained to me the subtleties of the experiments. Work could not have been done without the contribution of enthusiastic young collaborators: Kirk Green, Theodore Kolokolnikov, Michel Nizette, Didier Pieroux, Fabien Rogister, Marc Sciamanna, and Guy Van

der Sande. I am grateful to Gregory Kozyreff for his constructive criticisms of the manuscript and I thank my editor Achi Dosanjh who convinced me to go for an ambitious project. I acknowledge the Belgian National Science Foundation and the Pole Attraction Pole program of the Belgian government (2001-2006) for the support I received during the preparation of this book. Lastly I would like to thank Anne, Joan, and Marc for their love and support while I was away searching and computing. This book is dedicated to them.

Brussels, Belgium
January 2008

Thomas Erneux

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