Tensegrity Systems

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*To Judy, who inspires me,
and to my many students who dared to do something different*

Bob

To Dani, Bea, and Victor

Maurício

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Preface

The purpose of this book is to make contributions to the analytical machinery required to integrate structure and control design and to show that this optimized structure usually has a finite, rather than an infinite, complexity. The first challenge in such an endeavor is to choose the right paradigm for structure design. Engineers currently design wings for aircraft that coordinate the aerodynamics and structural design disciplines very well, but then the control functions are added, almost as an afterthought, destroying the flying efficiencies that were so carefully treated in the structure design in the first place. The added control functions are not efficiently performed either, given that the structure design has already been completed and cannot be economically redesigned to “cooperate” better with control functions. So, historically, engineers design structures first (beams, plates, and shells) and then add a control system to “torture” the structure into doing things it was not originally designed to do. Even in research communities we have not solved this problem of integrating structure and control design. As a recent research example of this fact, DARPA sponsored a “smart structures” research program to control wings by warping them (as the Wright Brothers did many years ago). An existing F16 wing was instrumented with a nickel–titanium actuator to apply a large torque to the wing to twist it, trying to obtain a 20° twist. The control system and the actuator twisted the wing only 7° because too much power was required to twist the structure away from its equilibrium. As opposed to a control system that torques or pushes the structure away from its equilibrium, we will show that a tensegrity paradigm for structures will allow one to *modify the equilibrium* of the structure to achieve the new desired shape, so that power is not required to hold the new shape. Integrating structure and control design will require less power from the control system to accomplish the same objectives.

Less control power also impacts the parameters of structure design, since less structural stress is imparted to the structural components during control. Such cooperation between the static and dynamic properties of the structure and the control system can only be accomplished by a structure design paradigm that maintains an extremely high degree of “controllability” during all phases of the structure design. This requires new types of

structures and the tensegrity structural paradigm is the only one the authors have found with these properties.

This book focuses on the development of analytical tools that would allow the dynamics and structure design to have the control problem in mind, but analytical tools first have to be derived for the static and dynamic analysis of the tensegrity paradigm, leaving room for only one chapter on control.

Even though this book focuses on *mechanical properties* of a tensegrity system, one need not limit design goals to merely mechanical properties. Other books will be written to optimize the geometric arrangement of material (within the tensegrity paradigm) to obtain a prescribed set of other *acoustic properties, or electromagnetic properties*. There are material properties that can be obtained by a “composite” design (where one chooses materials and the topology of their configuration) that natural materials do not possess. For example, we have demonstrated in the microwave lab a “left-handed” tensegrity material, meaning a material whose electromagnetic properties have negative values of permeability μ and permittivity ϵ .

Chapter 1, Introduction and Motivation, will give many small examples to illustrate the basic principles and the fundamental rules that govern tensegrity systems. Examples show tensegrity in nature (at both the nanoscale and the mesoscale), tensegrity in art, tensegrity in architecture, and tensegrity in engineering and science. This chapter is a guide to the book, referring to the types of problems to be addressed in more detail in later chapters and giving some of the optimization results that are simple to explain without the mathematical derivations that follow in later chapters. The rest of the book is fairly dense with mathematics, but Chapter 1 avoids math, to enlighten the spirit and intent and goals of the new methods.

Chapter 2, Analysis of Static Tensegrity Structures, provides the analytical tools to deal with the question *How many ways can sticks and strings be connected to yield a stable equilibrium?* This chapter finds the entire set of admissible member forces associated with any given configuration of the tensegrity system. The chapter also computes the stiffness matrix, predicts the stable configurations, and develops tools for kinematics.

Chapter 3, Design of Compressive Structures, is the first chapter on design of tensegrity systems. The chapter provides algorithms to minimize mass under compressive loads, subject to yield and/or buckling constraints, yielding minimal mass tensegrity structures. The molecular structure of the spider fiber, which is nature’s strongest tensile material, is indeed a class 1 tensegrity structure. Algorithms are given to design self-similar structures in an optimal way, to minimize mass, subject to buckling constraints. Formulas are also given to optimize the complexity of the structure, which turns out to be finite. After each additional self-similar iteration, the number of bars and strings increases, but, for a certain choice of geometry, the total mass of bars and strings decreases. This algorithm generates what we call *tensegrity fractals*. For certain structures, the string mass monotonically increases with

iteration, while the bar mass monotonically reduces, leading to minimal total mass in a finite number of iterations. The number of iterations required to achieve minimal mass is given explicitly in closed form by a formula relating the chosen geometry and the material properties.

Chapter 4, Design of Bending Structures, repeats the steps of Chapter 3, except for structures under bending loads. This chapter shows that the minimal mass structure, subject to bending loads, is a class 2 tensegrity structure. This work also shows how to optimize the complexity of a structure. The 1904 work of Michell derived the infinitely complex case, a continuum. Indeed, if mass at the construction joints is ignored, then the design of our optimal structure of finite complexity approaches the design of Michell, in the limit as the complexity of our design approaches infinity. If a mass penalty is associated with the presence of joints then the optimal complexity is finite.

Chapter 5, Analysis of Tensegrity Dynamics, derives the dynamics of tensegrity structures. Tensegrity concepts have been around for a long time, but in the absence of efficient algorithms to compute the dynamics, engineering designs were a rare novelty. This might explain the long gap between the invention of tensegrity as an art form and tensegrity as an engineering method. The large number of components, bars, and strings left many convinced that the required engineering would just be too complicated. But we must remind ourselves that “complication” is often measured by the relative ease with which the user of the analytical tools can do the design. Efficient new algorithms are now available to automatically, with minimal labor from the user, generate the equations of motion. This can be an enabler to engineering progress with tensegrity concepts in engineering and science.

NASA spent a lot of money in the 1960s and 1970s developing efficient algorithms for computer simulations of *multibody* systems. Each body in the system could be rigid or elastic, and many applications were forthcoming in spacecraft control, such as in SKYLAB. While these algorithms for generating dynamic equations of motion could indeed be used today for tensegrity systems, Chapter 5 will develop an even more efficient method for describing the dynamics. System constraints are added in the case of structures pinned to ground, or having fixed equal-length bars. This new result presents the dynamic equations in the form of a matrix differential equation in lieu of the traditional vector differential equations. Even though these equations are written using a non-minimal set of coordinates, efficiency comes from the very simple mathematical structure of the equations.

Chapter 6 will focus on methods for controlling tensegrity systems. Some recent results on the control of nonlinear non-minimal systems are derived to allow large motions of tensegrity systems. These methods are based upon Lyapunov stability theory. There are some new control-theoretic results here, but the main focus is to demonstrate the controllable features of tensegrity structures.

This text was designed for two different types of readers in mind. A person who wants the results without trudging through the proofs can read the chapters and skip the *Advanced Material* session at the end of each chapter. Those readers seeking a more rigorous treatment can read the chapter and may appreciate the more advanced material.

Finally, the authors would like to thank and acknowledge the many students and visitors that have been involved with their research on tensegrity in the past. This book mentions the work of Jack Aldrich, Joe Cessna, Waileung Chan, Carlos Cox, Bram de Jager, JeongHeon Han, Milenko Masic, Tino Mingori, Jean-Paul Pinaud, Cornel Sultan, Jeff Scruggs, Carlos Vera, Darrell Williamson, and Anders Wroldsen, among others.

Throughout the text, many plots and figures are better understood and appreciated in color. We tried to generate these figures in such a way that the most important information could be captured from the black-and-white prints of this book. For completeness however, in many points of this book, the reader might encounter references to colors in the figures. In this case, please refer to the full color version of these figure which are available in the website (www.springer.com) or on the electronic version of this book.