Graduate Texts in Mathematics 242

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Abstract Algebra

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Preface

This book is a basic algebra text for first-year graduate students, with some additions for those who survive into a second year. It assumes that readers know some linear algebra, and can do simple proofs with sets, elements, mappings, and equivalence relations. Otherwise, the material is self-contained. A previous semester of abstract algebra is, however, highly recommended.

Algebra today is a diverse and expanding field of which the standard contents of a first-year course no longer give a faithful picture. Perhaps no single book can; but enough additional topics are included here to give students a fairer idea. Instructors will have some flexibility in devising syllabi or additional courses; students may read or peek at topics not covered in class.

Diagrams and universal properties appear early to assist the transition from proofs with elements to proofs with arrows; but categories and universal algebras, which provide conceptual understanding of algebra in general, but require more maturity, have been placed last. The appendix has rather more set theory than usual; this puts Zorn's lemma and cardinalities on a reasonably firm footing.

The author is fond of saying (some say, overly fond) that algebra is like French pastry: wonderful, but cannot be learned without putting one's hands to the dough. Over 1400 exercises will encourage readers to do just that. A few are simple proofs from the text, placed there in the belief that useful facts make good exercises. Starred problems are more difficult or have more extensive solutions.

Algebra owes its name, and its existence as a separate branch of mathematics, to a ninth-century treatise on quadratic equations, *Al-jabr wa'l muqabala*, "the balancing of related quantities", written by the Persian mathematician al-Khowarizmi. (The author is indebted to Professor Boumedienne Belkhouche for this translation.) Algebra retained its emphasis on polynomial equations until well into the nineteenth century, then began to diversify. Around 1900, it headed the revolution that made mathematics abstract and axiomatic. William Burnside and the great German algebraists of the 1920s, most notably Emil Artin, Wolfgang Krull, and Emmy Noether, used the clarity and generality of the new mathematics to reach unprecedented depth and to assemble what was then called modern algebra. The next generation, Garrett Birkhoff, Saunders MacLane, and others, expanded its scope and depth but did not change its character. This history is documented by brief notes and references to the original papers. Time pressures, sundry events, and the state of the local libraries have kept these references a bit short of optimal completeness, but they should suffice to place results in their historical context, and may encourage some readers to read the old masters.

This book is a second edition of *Algebra*, published by the good folks at Wiley in 1999. I meant to add a few topics and incorporate a number of useful comments, particularly from Professor Garibaldi, of Emory University. I ended up rewriting the whole book from end to end. I am very grateful for this chance to polish a major work, made possible by Springer, by the patience and understanding of my editor, Mark Spencer, by the inspired thoroughness of my copy editor, David Kramer, and by the hospitality of the people of Marshall and Scottsville.

Readers who are familiar with the first version will find many differences, some of them major. The first chapters have been streamlined for rapid access to solvability of equations by radicals. Some topics are gone: groups with operators, Lüroth's theorem, Sturm's theorem on ordered fields. More have been added: separability of transcendental extensions, Hensel's lemma, Gröbner bases, primitive rings, hereditary rings, Ext and Tor and some of their applications, subdirect products. There are some 450 more exercises. I apologize in advance for the new errors introduced by this process, and hope that readers will be kind enough to point them out.

New Orleans, Louisiana, and Marshall, Texas, 2006.

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