

Applied Mathematical Sciences

Volume 162

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(continued following index)

Habib Ammari Hyeonbae Kang

Polarization and Moment Tensors

With Applications to Inverse Problems and
Effective Medium Theory

With 25 Figures

 Springer

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ISBN-13: 978-0-387-71565-0

e-ISBN-13: 978-0-387-71566-7

Printed on acid-free paper.

Library of Congress Control Number: 2007925444

Mathematics Subject Classification (2000): 35R30, 35B27, 35B40, 35R05, 74B05, 78M40, 78M35

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9 8 7 6 5 4 3 2 1

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Preface

Recent developments in imaging and effective medium theory reveal that these fields share several fundamental concepts. One of the unifying threads is the use of generalized polarization (GPTs) and moment tensors (EMTs) that depend only on the geometry and the conductivity or the Lamé parameters of the inclusion. These concepts generalize those of classic Pólya–Szegő polarization tensors, which have been extensively studied in the literature by many authors for various purposes. The notion of Pólya–Szegő polarization tensors appeared in problems of potential theory related to certain questions arising in hydrodynamics and in electrostatics.

The study of GPTs and EMTs with applications in imaging and effective medium theory forms the heart of the book. We show that GPTs and EMTs are key mathematical concepts in effectively reconstructing small conductivity or elastic inclusions from boundary measurements as well as in calculating accurate, effective electrical or elastic properties of composite materials.

Due to the character of its topic, this book is of interest not only to mathematicians working in inverse problems and effective medium theory, but also to physicists and engineers who could communicate with mathematicians on these issues. It highlights the benefits of sharing new, deep ideas among different fields of applied mathematics.

This book would not have been possible without the collaborations and the conversations with a number of outstanding colleagues. We have not only profited from generous sharing of their ideas, insights, and enthusiasm, but also from their friendship, support, and encouragement. We feel especially indebted to Graeme Milton, Gen Nakamura, Jin Keun Seo, Gunther Uhlmann, and Michael Vogelius.

Paris and Seoul,
March 2007

Habib Ammari
Hyeonbae Kang

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