Part I
Theory of Equations Over Languages and Automata

In this part we present the basics of equations over languages and automata, and specialize the theory to finite and ω-regular languages.
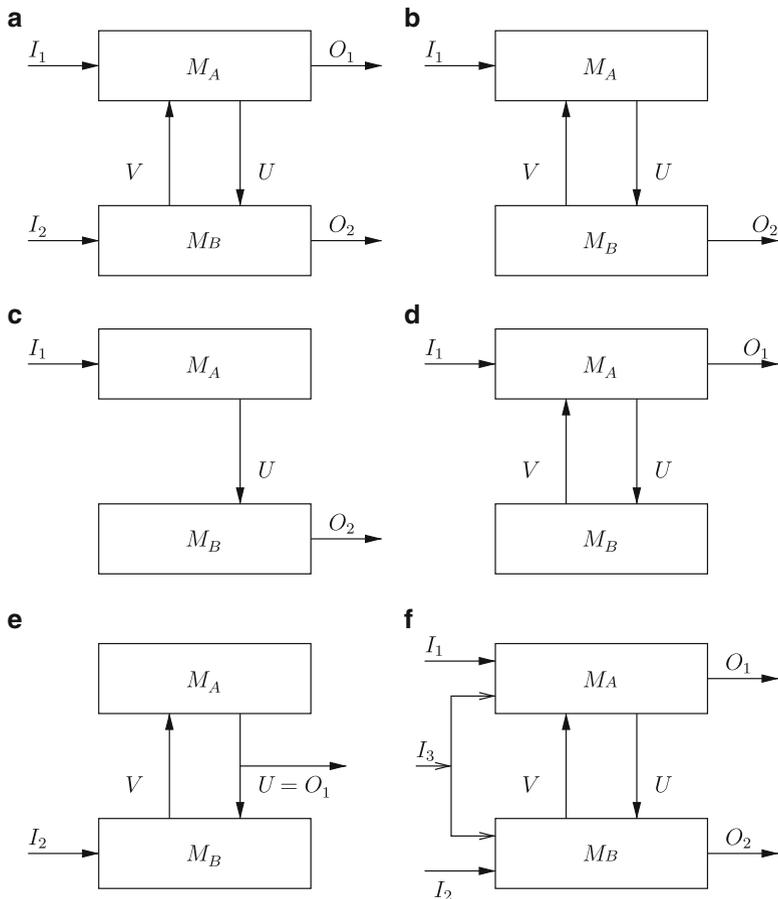
In Chap. 2 we define and study *abstract equations over languages*, to obtain results valid for any language equation. We investigate two composition operators for abstract languages: synchronous composition, $\bullet$, and parallel composition, $\diamond$, and we check conformity by language containment.

A key contribution is the *computation of the most general solutions of the language equations* $A \bullet X \subseteq C$ and $A \diamond X \subseteq C$, found respectively as $S = \overline{A \bullet \overline{C}}$, and $S = \overline{A \diamond \overline{C}}$. The derivation sheds lights on the properties required of a composition operator to yield such a closed formula as the largest solution, and explains when different equations give rise to that same type of solution formula. These formulas turn out to subsume a panoply of specialized solutions derived in the past for specific composition operators and topologies. Some common network topologies are shown in Fig. 1.1.

Then in Chap. 3 we specialize language equations to languages associated with classes of automata used for modeling hardware and software systems, namely, regular languages as counterparts of finite automata, FSM languages as counterparts of FSMs. Thus we can operate algorithmically on those languages through their automata and study how to solve effectively their related language equations. It is important to find solutions within the same language class of the equation, e.g., when studying FSM language equations we look for solutions that are FSM languages. Moreover, we are interested in subsets of solutions characterized by further properties of practical interest, e.g., FSM languages that satisfy the Moore property; thus the solutions are restricted further.

In Chap. 4 we study the extensions to equations over ω-automata of the results obtained in the finite regular case.

Various contributions, investigating partial aspects of the topic of this research, have been published. A complete survey is provided in Chap. 5.

a

I_1 → [M_A] → O_1

V U

I_2 → [M_B] O_2 →

b

I_1 → [M_A]

V U

[M_B] O_2 →

c

I_1 → [M_A]

U

[M_B] O_2 →

d

I_1 → [M_A] → O_1

V U

[M_B]

e

[M_A]

V $U = O_1$

I_2 → [M_B]

f

I_1 → [M_A] → O_1

I_3 →

V U

→ [M_B] O_2 →

I_2

Fig. 1.1 Patterns of composition. (**a**) general topology; (**b**) 2-way cascade (or 2-way series) topology; (**c**) 1-way cascade (or 1-way series) topology; (**d**) rectification topology; (**e**) controller's (or supervisory control) topology; (**f**) variant of general topology