Parallel Coordinates

Visual Multidimensional Geometry and Its Applications

To

Hadassah, Dona and Haim, Avigail, Ofra, Yuval, Akiva
... a wonderful family

and

Bernie Dismdale, Lew Leeburg, Alex Hurwitz,
Misha Boz, Baxter Armstrong, Ralph Alterowitz
... great friends and colleagues

and

Heinz von Foerster
... Mischiefologist, Magician,
Cybernetician, and Teacher

Parallel Coordinates

Visual Multidimensional Geometry
and Its Applications

with 230 color illustrations

Alfred Inselberg

Foreword by Ben Shneiderman

CD-ROM

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Foreword

The breakthrough idea of parallel coordinates has enchanted many people with its cleverness and power. Al Inselberg, the source of that cleverness and power, finally shares the depth and breadth of his invention in this historically important book.

I've been waiting impatiently for this exposition since I was captivated by Inselberg's lecture at the University of Maryland in 1979. He had already understood the potential parallel coordinates has for generating insights into high-dimensional analysis and information visualization. In the following decades he polished the arguments, built effective software, and demonstrated value in important applications. Now a broad community of readers can benefit from his insights and effective presentation.

I believe that Inselberg's parallel coordinates is a transformational ideal that matches the importance of René Descartes' (1596–1650) invention of Cartesian coordinates. Just as Cartesian coordinates help us understand 2D and 3D geometry, parallel coordinates offer fresh ways of thinking about and proving theorems in higher-dimensional geometries. At the same time they will lead to more powerful tools for solving practical problems in a wide variety of applications. It is rare to encounter such a mind-shattering idea with such historic importance.

While Inselberg's insight and exposition opens the door to many discoveries, there is much work to be done for generations of mathematicians, computer scientists, programmers, and domain experts who will need to build on these innovative ideas.

To understand and apply parallel coordinates, many further innovations are needed. Those who can readily reshape their perceptual skills and realign their cognitive frames will be well-equipped for this nontrivial task. I still struggle with these novelties, which Inselberg has masterfully accomplished. In each of my dozen meetings with Inselberg, he relieved me of my confusion and steered me closer to clarity. There is a chance that those who learn parallel coordinates early in life may be able to more easily see higher-dimensional spaces. More likely, master teachers, inspired interface designers, and gifted domain experts will enable future generations to grasp these profound ideas as if they were entirely natural, just as we see Cartesian coordinates.

I encourage readers to read each paragraph carefully and make sure they fully grasp the ideas. Inselberg has made this possible by his lucid and often charming prose, filled with intriguing historical references and clever quips. The computer scientists, programmers, dataminers, and information visualization experts may want to jump ahead to Chapter 10 which has practical examples and numerous screen shots. The recent additional insights from Inselberg's students and followers give a hint of the thousands of papers that are likely to be inspired by this work. More importantly, the applications of parallel coordinates could contribute to solving some of the complex multidimensional problems of our time. I hope every reader will strive to put these ideas to work in making a better world.

Ben Shneiderman
University of Maryland, College Park
May 2008

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Preface

What and Why

In the late 1980s, researchers in computer graphics recognized a specific stream of applications as an emerging field and called it *visualization*. By now it has become a discipline with its own journals, conferences, and community of active workers applying it to science, city planning, entertainment, and much more. Even our reality is becoming more virtual. Yet since time immemorial people have expressed themselves visually, realizing in due course insight can also be *derived* from images [63]. In 1854, a cholera epidemic raging in a London neighborhood prompted Dr. John Snow to search for remedies. He examined the data consisting of a table with the addresses of those who had died from the disease. On a map of the neighborhood, which fortunately had the positions of the water wells, he placed dots at the locations of the recorded deaths. The concentration of dots in the vicinity of just one well was *visually* striking. He became convinced that there was a connection, and had the handle of the suspect well replaced. The epidemic stopped! Apparently the disease was being transmitted by contact with the handle. This true story is widely considered an early success of visualization [172]; the picture provided insight no one had gleaned from the table of the data.

Legend has it that Archimedes was absorbed in a diagram when a Roman soldier killed him; the first recorded death in defense of visualization! “Do not disturb my circles” he pleaded as he was being struck by the sword. Visualization flourished in geometry, where visual interaction with diagrams is interwoven in the testing of conjectures and construction of proofs. Our tremendous ability in pattern recognition enables us to extract insight from images. This essence of visualization is abstracted and adapted in the more general problem-solving process to the extent that we form a mental image of a problem we are trying to solve and at times we say *see* when we mean understand.

My interest in visualization was sparked and nourished while learning Euclidean geometry. Later, while studying multidimensional geometries I became frustrated by the *absence* of visualization. Basically, we were working with equations, interpreting them geometrically, without the enjoyment and benefit of pictures. I kept wondering about ways to draw “pictures” of multidimensional objects. Of course, it would also be great fun doing *synthetic* high-dimensional constructions and have tools for accurate visualization of multivariate problems. These thoughts, best attributed to the impetuousity of youth, absorbed me. What coalesced from this struggle is *parallel coordinates*.

A superficial resemblance to nomography [19] was noted early in the development [97]. "Nomograms are drawings made on the plane in order to replace cumbersome numerical calculations" ([144] p. 9), a technique that declined with the advent of modern computation. I was unaware until recently[2] of d'Ocagne's marvelous monograph on *parallel coordinates* in two dimensions [141] with a *point* ↔ *line* correspondence. D'Ocagne was interested in the computational applications of nomography [142] rather than the development of a multidimensional coordinate system, which is where we come in. Invented independently, *parallel coordinates is a general-purpose VISUAL multidimensional coordinate system*. There are theorems on the unique representation of multidimensional objects, and geometrical algorithms for intersections, containment, minimal distances, proximities, etc., with a wide variety of applications.

The writing is undertaken with trepidation, knowing what kind of book it should not be: not a text in mathematics, though it should contain the foundations and development, nor computer science nor the other components of the methodology, nor a source-book for the variety of included applications. Rather it is an amalgam of all these, comprehensible and appealing to mathematicians, computer scientists, statisticians, engineers, scientists, and all those wanting to acquire deeper understanding of their multivariate problems by way of visualization.

In due course, it is hoped that the methodology will not only find its way into textbooks on various applications, but also be adopted for *visual experimentation and explanations* in teaching mathematics and the sciences at the university, high-school, and even elementary-school levels.

Style and Structure of the Book

This is a textbook with ample material for a one-semester course, easily adapted to shorter periods (see the following section), and containing advanced seminar topics. The writing evolved during a decade of teaching "visualization of multidimensional geometry and its applications," a popular elective course in the applied mathematics/ computer science curricula. It is also a reference for self-study or a companion text for courses on information visualization (particularly the chapter on data mining), visualization in general, data mining, GIS, management, statistics, linear algebra, analytic geometry, complex variables, and fields dealing with multivariate problems

[2] I am indebted to M. Friendly for pointing this out in [63].

(economics, engineering, physics, psychology, management, arts, and social sciences). The parallel coordinates methodology provides a multidisciplinary visual language and paradigm.

The mantra is to let intuition guide the formalism starting with the motivation for the emergence of parallel coordinates (∥-coords). Then the 2-dimensional plane is viewed with ∥-coords as compared to Cartesian coordinates, leading to the discovery of surprising and pleasing properties. The chapters on the representation of lines, hyperplanes, curves, and topologies for proximity of lines and hyperplanes are interlaced with applications on collision-avoidance algorithms for air traffic control, geometric modeling, computer vision, statistics, and more. On this foundation the representation of surfaces is built, yielding exciting results such as viewing *convexity in any dimension*, recognizing nonorientability (as for the Möbius strip), and applications to intelligent instrumentation, process control, and decision support. There follows an *easy-to-read chapter on data mining and information visualization* with many fun applications (exploratory data analysis (EDA) on satellite, financial, vehicle recognition from noise-signature, astronomical and industrial datasets, automatic classification, decision support, nonlinear models). Results too recent to be incorporated in the text are included in the appendix at the end. They complement the representation of lines, developable surfaces, visualization and analysis of large networks, detecting and separating clusters of coplanar points, and the visualization of complex-valued functions: *to see* $\mathbb{C}^2$. The emphasis is on *visualization*, though computational and analytical aspects are treated rigorously, and where needed, extensively. Each chapter contains a variety of exercises, easy ones at first to check the understanding of the basics and more with increasing difficulty. Some exercises have served well as semester project topics. Where appropriate, open questions and research directions are pointed out. In places, details are deferred to references when technicalities may obscure the underlying ideas or unnecessarily intimidate the reader. I hope that the more mathematically sophisticated readers will forgive the occasional "sleights of hand."

To make the book reasonably self-contained, after the introduction, there is a chapter on the needed geometrical background. It covers projective geometry, explaining the notions of *duality* and *homogeneous coordinates* as used in the subsequent rigorous development. The application to perspective, as in computer graphics, is developed with the realization that the "points at infinity" on the perspective plane are images of ideal points (directions). There is also a short intuitive chapter on the theory of envelopes; helpful background for the ensuing chapters on curves and surfaces. The attached CD contains the *interactive learning module* (ILM) for self-study and class demonstrations. There are pointers and guidance for its use at the relevant sections in the text. Projective geometry theorems, dualities, transformations, and properties of representations are understood quickly and enjoyably.

♣ FT Fast Track: How to Skip Reading Most of the Book

In each chapter the key portions are marked successively by ♣ *FT-5*. The numeral, here 5, indicates the fifth fast-track marker within the chapter. The essentials can be swiftly grasped by following these road signs and returning for additional details later if needed. Typically the ♣ *FT-5* symbol is placed at the beginning of a section to be read to the end or up to an "end marker" such as ♣ *FT-5e*. The landmarks[3] may also be figures. Covering these highlights is also a good way to review the material. The fast-track entries appear in the table of contents for sections or subsections. Portions marked with ** contain in-depth treatment of specific topics and are not needed for the basic ideas and results. Those interested in the applications can read the chapter on data mining and then scan the text for more details. Otherwise, choose your goal and follow the illuminated path:

- **the essentials can be mastered quickly with the FT option;**
- **for extensive and sound foundations, include unmarked portions;**
- **to reach expert level, add some ** sections.**

Course Organization

Much has been learned from the teaching experience. Reasonably well prepared and motivated students enjoyed and absorbed the material readily, contributing new results and ideas. After some experimentation, the course outline that proved most successful is the presentation of a course overview in the first two to three hours, including some demonstrations. This motivates the subject with problems and examples and also shows why, due to the 2-D point $\leftrightarrow$ line duality, parallel coordinates need to be considered in the projective rather than the Euclidean plane. The development as given in the chapter sequence can then be safely followed. Demonstrations with the *ILM* in the CD as well as several applications are always well received. The instructor can organize group projects that provide such demonstrations. Some of the ones already given are available over the Internet. Also, there is an increasing collection of software commercially or freely available that is suitable for class demonstrations and use.

[3] I am indebted to Chao-Kuei Hung for this wonderful idea.

Instead of exams, the students, usually in groups of two or three, do projects. They are encouraged to choose topics in some area in which they have a personal interest and apply what they have learned in the course. Such projects have involved expanding the mathematical foundations, development and implementation of algorithms, writing programs particularly for data mining and modeling, and more. Examples of actual projects include "visualization of the Internet," "collision detection and intersection in billiards," "visualizing the 'learning' process in neural networks," "topologies for proximity of flats (higher-dimensional planes)," several geometrical construction algorithms, "characterizing painters (i.e., impressionists, etc.)," "characterizing music," data mining on a great variety of multivariate datasets (i.e., pollutants, bioinformatics, weather data, computer dating, constituents of various medicines and foods, etc.), differentiation and integration, fractals and linear programming with ‖-coords, large network visualization, visualizing complex-valued functions, and other topics. A wide variety of software on representations of surfaces, curve-plotters, implementation of results (i.e., translations $\leftrightarrow$ rotations), etc. were produced. Some projects were subsequently presented at conferences, and others led to master's theses. Student are encouraged to pursue self-discovery, including writing programs for experimentation and testing of conjectures. Solutions to many exercises are included at the end. The recommended course syllabus follows.

** Syllabus

- Introduction to scientific information and multidimensional visualization, complete course overview. One lecture with demos on the foundations and numerous applications.
- Projective geometry: foundations, dualities, collinearity theorems, homogeneous coordinates, linearization of projective transformations, conics in $\mathbb{P}^2$, applications to computer graphics: transformations and their matrices; perspective: points at infinity are images of *ideal points*, geometrical invariants. Two lectures with demos.
- Parallel coordinates in the plane: dualities: *point* $\leftrightarrow$ *line*, *translation* $\leftrightarrow$ *rotation*, other transformations, parallelism and orthogonality, visual and automatic data mining. One lecture with demos.
- Lines in N-space; two lectures
  - Representing lines by $(N-1)$ points with two indices, parallel lines, general representation, the 3-point collinearity property; demos.

  - Transformations, improving the display of multidimensional lines, intersections, minimum L_1 distance between pairs of lines, upper and lower proximity bounds.
  - Application: collision-avoidance algorithms for air traffic control; demos.

- Planes, hyperplanes, and flats in N-Space; three lectures.

  - Detecting coplanarity of points on a grid, representing hyperplanes by $(N - 1)$ parallel lines and a point (polygonal line), duality: *rotation of a plane about a line* $\leftrightarrow$ *translation of a point on a line*, manufacturing data; demos.
  - Recognizing M-dimensional objects from their $(M - 1)$-dimensional subsets, *recursive construction mapping*.
  - Detecting coplanarity of randomly chosen points, representing hyperplanes by $(N - 1)$ points with N indices, the family of *superplanes* and their special properties, reading the equation of a hyperplane from the picture, duality: *rotation of a plane about a line* $\leftrightarrow$ *translation of* $(N + 1)$ *points on lines*, designing rotations and transformations from their "planar signature," synthetic constructions: parallel planes, intersections, above/below relation between points and planes and more, "near coplanarity," separating point-clusters on several planes; demos.

- Envelopes of one-parameter families of curves, background for the curves and surfaces chapters. One-half lecture (discretionary).
- Curves; one and one-half lectures.

  - Point-curves and line-curves, convex down/up curves and their transforms, duality: *cusp* $\leftrightarrow$ *inflection point*, *point-curve* $\leftrightarrow$ *line-curve*.
  - *conics* $\leftrightarrow$ *conics* via the Möbius transformation; six cases, algrebraic curves: the Plücker formulas and dualities, curve plotters, examples of curve transforms, analysis of important points and portions of curves, periodicities, and symmetries.
  - Convex curves and their relatives, generalized conics and their dualities; six cases, operational transformations.

- Families of proximate hyperplanes; one and one-half lectures.

  - Topology for proximity of flats.
  - Proximity of lines and line neighborhoods.
  - Proximity of hyperplanes, formulation of the problem in N-space, hyperplane neighborhoods with $(N - 1)$ regions, the three cases: bounded

convex polygons, unbounded convex polygons, generalized hyperbolas, constructing neighborhoods with complexity $O(N)$, the general case.
- Navigation within neighborhoods, finding the $(N-1)$ points representing specific hyperplanes, matching algorithm; demos.
- Applications, a central problem in many fields, statistics (regression), geometric modeling, computer vision.

• Hypersurfaces in N-space; two lectures.

  - Formulation of the problem, hypersurfaces represented by $(N-1)$ *linked* regions, boundary contours, quadric and general algebraic hypersurfaces.
  - Developable hypersurfaces represented by $(N-1)$ linked curves, ambiguities and uniqueness, reconstruction, classes of developables, duality: *space curves* $\leftrightarrow$ *developables*, examples: developable helicoids and more; demos.
  - Ruled hypersurfaces, several examples of their representation including Möbius strip, visualizing nonorientability in N dimensions; demos.
  - More general hypersurfaces, convex and nonconvex, and how to distinguish them in N dimensions, searching for matching algorithms, interior points.
  - An important approximate representation, interior points, applications to decision support and intelligent process control; demos.

• Data mining and other applications (self-contained chapter); two lectures.

  - Origins, the case for visualization.
  - exploratory data analysis (EDA); visual data mining

    * Multidimensional detective, atomic queries, an easy case study: GIS data, Boolean operators and compound queries; financial data, hundreds of variables; demos.

  - Automatic classification, feature selection and ordering, comparisons, and two case studies; demos.
  - Visual and computational models, decision support: interelationships, sensitivies, constraint and trade-off analysis; demos.

• Recent developments (appendix); two lectures (discretionary).

  - Representing several lines efficiently; Shlomi Cohen-Ganor.
  - Separating points on several planes; Nir Shahaf.
  - Surface representation decomposition and developable surfaces; Chao-Kuei Hung.

  - Network visualization and analysis with ∥-coords; Yaron Singer and Ohad Greenspan.
  - To see $\mathbb{C}^2$: visualizing complex-valued functions; Yoav Yaari.

- Time permitting, students can choose to lecture on various topics in visualization, e.g., Internet visualization, fluid flow, visualizing the function of neural networks, detecting network intrusions, etc.

Prerequisites

The course attracted students from mathematics, computer science, engineering, statistics, management, data mining, and a wide variety of sciences (physics, chemistry, biology, etc.), geography, social sciences, and even linguistics. They ranged from second year to advanced doctorate students. Background in linear algebra is important for those who want to master the subject. Programming skills including some computer graphics are helpful though not necessary.

Acknowledgments

I first proposed parallel coordinates in 1959 when I was a graduate student at the University of Illinois (Champaign-Urbana). Two topologists, Professors D. Bourgin and S.S. Cairns, encouraged me to pursue the idea, but it took me many years to take their advice seriously. Later, I had the pleasure and priviledge to collaborate with Bernie Dimsdale[4] at the IBM Los Angeles Science Center (LASC), who made many important contributions and left his imprint throughout this work. Also at LASC, Alex Hurwitz's numerous contributions, suggestions, and ideas as well as Juan Rivero's application to the phase space for nonlinear differential equations benefited the early development.

From the students, starting in 1978 Moti Reif was bravely the first to do his graduate research on parallel coordinates (convexity algorithms) [110]; John Eickemeyer discovered among other things the "superplanes" [47]; Avijit Chatterjee developed Eickemeyer's results further for the visualization of polytopes in N-space [23]; Misha Boz made many contributions over the years, and together with Bernie Dimsdale, we did the first successful implementation of the collision avoidance algorithms [104]; Steve Cohan [31] and Paolo Fiorini developed applications to robotics [60]; Jim Adams (circa 1981), Alex Hurwitz, and later Tuval Chomut pioneered the first successful implementation of parallel coordinates for exploratory data analysis (EDA) [30] as well as contributing to the convexity algorithms with Moti Reif; Chao Kuei Hung derived an elegant characterization of developable surfaces in terms of their tangent planes [93], [108] (see [94] for updated results); Lou Ann Colby investigated dualities between rigid transformations; Richard Addison later improved the implementation of the collision avoidance algorithms; and Greg Wong together with John Eickemeyer under Alex Hurwitz's leadership developed and implemented further substantial improvements; Tova and Shlomo Avidan made innovative improvements on the data mining application [5], [103]; Tanya Matskewich and Yaakov Brenner did a very sophisticated and useful characterization of proximity for flats [131] that developed into a full chapter; Nadav Helfman [81] further developed the application to decision support; Adi Shlomi designed and wrote much of the software in the *ILM*. Liat Cohen solved the exercises and also contributed many ideas and improvements. David Adjiashvili wrote the

[4] A long-term associate of John von Neuman.

beautiful software displaying surfaces in ‖-coords. In the spirit of geometry, these pictures led to key results that were subsequently proved. Several other student projects made important innovations and are acknowledged in the text.

Many researchers contributed to the development and applications of parallel coordinates (in alphabetical order). The Andrienkos [2], J. Dykes et al. [44], R. Edsall [45], and A. MacEachren et al. [43] introduced ‖-coords to GIS and geovisualization, Kim Esbensen [53] contributed to data analysis and dualities, H. Carter, C. Gennings, and K. Dawson on response surfaces in statistics [67]; Amit Goel [71] on aircraft design; Chris Jones in optimization [115]; John Helly [82] on early application to data analysis; Hans Hinterberger's [156] contributions in comparative visualization and data density analysis; Matt Ward et al. introduced hierarchical parallel coordinates [65] and added much to the field [181], Helwig Hauser's innovative contributions included parallel sets for categorical data [79]; Antony Unwin et al. made wide-ranging numerous contributions [177]; Li Yang [187] applied ‖-coords to the visualization of association rules; H. Choi and Heejo Lee's [29] and P. Hertzog [83] are important contributions to detecting network intrusion, other important contributors to the field include Luc Girardin and Dominigue Brodbeck [70], Xiaxin Yen, et al. [189], T. Itoh, et al. [111], and Stefan Axelsson [6]. G. Conti [32] produced a ground-breaking book on security data visualization; there are exciting recent works by H. Ye and Z. Lin's [188] with an innovative contribution to optimization (simulated annealing); T. Kipouros et al. [120] with a sophisticated optimization for turbomachinery design [120]; S. El Medjani et al. [49] proposed a novel application using ‖-coords as straight-line detectors; R. Rosenbaum and H. Schumann [151] on progression in visualization; F. Rossi [153] on visual data mining and machine learning; Huamin Qu et al. [89] on air pollution analysis and clustering; M. Tory et al. on ‖-coords interfaces [171]; J. Johanson et al. [113]; G. Ellis and A. Dix [50] on clustering and clutter reduction; H. Siirtola and K.J. Räihä on interaction with ‖-coords; B. Pham, Y. Cai and R. Brown on traditional Chinese medicine [147], and there are proposals to enhance ‖-coords using curves [129], place them in 3-D [139], [114], or modify them as starplots [51]. C.B. Hurley and R.W. Olford contributed to the definitive study on axes permutations [95]. This list is by no means exhaustive.

There have been a number of noteworthy visualization contributions since the 1980s: the grand tour by Asimov [4], dimensional inference and more by Buja et al. [66], multidimensional scaling and convergence of correlation matrices by Chu-Hou Chen [27], [26], a chain of valuable and varied ideas (most notably RadViz) by Georges Grinstein et al. [87] [88] and Daniel Keim et al. [119] [118], hierarchical visualization of multivariate functions by Mihalisin et al. [135], [84]. Eminent are the seminal contributions of Jacob Zaavi and Gregory Piatetsky-Shapiro et al. [57] in all aspects of data mining and the role of visualization.

Bertin's book [14] is the early classic in *information visualization*. The field has been blossoming since then as attested by the volumes [22] and [11] containing organized collections of important papers, the seminal source on data mining [57], and the wonderful texts of Colin Ware emphasizing perception and design [182], Chaomei Chen's [25] work on a range of cognitive social and collaborative activities, and the comprehensive expository collection of methodologies by Robert Spence [164].

I have been supported and introduced to a variety of good problems by several friends and colleagues, especially (listed in alphabetical order) by George Bekey, Ronald Coiffman, Julia Chung, Alon Efrat, Marti Hearst, Peter Jones, Pei Ling Lai [109], [124], Khosrow Motamedi, Karl Müller, David McIlroy, Niko Shlamberger and Ana Trejak, Randall Smith of GM Research, and Fred Warner. I am especially grateful to John Nacerino of EPA in Las Vegas for his guidance and steadfast encouragement. His initiative and efforts enabled the distribution of the parallel coordinates software to academic researchers. Since 1990 the IEEE annual visualization conferences have been an excellent forum in which to learn and present ideas as well as receive valuable suggestions and criticism. In this community the following colleagues were especially helpful: Dan Bergeron [185], Ed Chi [28], Georges Grinstein, Beth Hetzler, Helwig Hauser, Daniel Keim, Chaim Lefkovits, Ted Mihalisin, Tamara Munzer, Alex Pang, Catherine Plaisant [160], Bill Ribarsky, Bernice Rogowitz, Holly Rushmeier, Lloyd Treinish, Bob Spence and Lisa Tweedie [175], John Stasko, Theresa-Marie Thyne, Matt Ward, and Pak C. Wong, who together with R.D. Bergeron wrote the first survey of the field [185] among their other contributions. I learned a great deal from Ben Shneiderman's [159] [160], [11] fundamental insights and encyclopedic contributions to visualization, and also from Marti Hearst's [112], [80] courses, varied interests and stimulating ideas in information visualization, user interfaces, and text mining. Of great benefit has been the interaction with the statistics and statistical graphics communities, especially with Daniel Asimov, Andreas Buja, Jogesh Babu, Dianne Cook [34], Larry Cox, Daniela Di Benedetto [33], Michael Friendly [62], Heike Hoffman, Catherine Hurley, Moon Yul Huh [91], Sigbert Klinke, Carlo Lauro [125], Jungi Nakano [139], Francisco Palumbo et al. [125], David Scott, Valentino Tontodonato [170], Ritei Shibata et al. [123], Deborah F. Swayne [168], Martin Theus, Luke Tierney, Simon Urbanek, Antony Unwin [176], Bill Venables, Ed Wegman [184], Adalbert Wilhelm, Lee Wilkinson and Graham Wills.

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