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# Soft Computing For Complex Multiple Criteria Decision Making

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 Springer

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Library of Congress Control Number: 2005935530

ISBN-13: 978-0387-30243-0 (HB)    ISBN-13: 978-0387-30177-8 (e-book)  
ISBN-10: 0-387-30243-3    (HB)    ISBN-10: 0-387-30177-1 (e-book)

Printed on acid-free paper.

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Printed in the United States of America.

9 8 7 6 5 4 3 2 1

SPIN 11052739

[springer.com](http://springer.com)

*To  
Ewa.*

*To  
Dominika,  
Kacper,  
Agnieszka.*

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# Preface

*"Daprima importa sapere di che cosa si tratta."  
("The first thing is to know what the talk is about".)*

Attributed to **Vilfredo Pareto**.

This book results from my continuous and deep interest in multiple criteria decision making (MCDM). Eleven years ago I wrote in my previous monograph: *"This work results from my interest in the field of vector optimization. I stumbled first upon this subject in 1982 [ ... ]. I was attracted then by a gap between vector optimization used to serve as a formal model for multiple objective decision problems and the decision problems themselves, the gap nonexistent in scalar optimization. Roughly speaking, vector optimization provides methods for ranking decisions according to a partial order whereas decision making requires a linear ordering of decisions."* This declaration is still valid and nothing needs to be changed.

To be more specific, this book is a fruit of my dissatisfaction with the current state-of-the-art of MCDM. MCDM is a branch of science, whose declared ultimate goal is to provide practical tools. However, we cannot say, and this is regrettable, that all present MCDM methods and algorithms are in popular use by those who make complex decisions and for that purpose are in need of methodological or computational support. And this is despite the number of scientists involved in research in the field, thought to be well over one thousand persons active worldwide, and the cumulative number of relevant publications, assessed to be in a range

of thousands. I have acquired a sharper view on the matter since when holding, in addition to my scientific affiliation, some advisory positions in business.

It seems that the key word to understand the situation is "involvement". Moreover, it is the complexity of interfaces between users and MCDM methods rather than the involvement of the methods that is to be blamed for limited MCDM popularity in real-life decision making. To illustrate and to put in a broader perspective what I mean by an interface let me recall the famous Black-Scholes close-end formula for pricing stock options. The methodology behind the Black-Scholes development is involved indeed and calls for knowledge of advanced stochastic calculus. However, the interface - in this case the formula itself - can be easily programmed and calculated in a spreadsheet or in a pocket calculator with financial built-in functions. This is how knotty mathematics can become accessible to a wide spectrum of lay users.

Though the notion of close-end formulas is not relevant to MCDM, lay and broad use of MCDM will be not possible without simple, low computing-intensive methods. This, I claim, is the main challenge in MCDM for the coming years.

This book is a modest step towards meeting this challenge. When writing it my governing rule was to use only as much formalism as can be immediately consumed in practical schemes and support tools.

The book summarizes my research performed in the Systems Research Institute of the Polish Academy of Science in the period after the publication of my first monograph on MCDM in 1994.

The title for the book was selected, on purpose, to be slightly intriguing. I hope, however, that the contents of the book fully justifies it.

*Warsaw, June 2005.*



# Notation

- $\mathcal{R}^k$  –  $k$  – dimensional real space ( $\mathcal{R}$  for  $k = 1$ ),
- $\mathcal{X}, \mathcal{Y}$  – spaces,
- $A, \dots, Z$  – sets,
- $R_+^k$  – nonnegative orthant of  $\mathcal{R}^k$ ,
- $x, y$  – elements of a space or a set,
- $x_i, y_i$  –  $i$  – th component of elements  $x, y$ ,
- $\{x, y, \dots\}$  – set composed of elements  $x, y, \dots$ ,
- $\emptyset$  – empty set,
- $\subseteq$  – set inclusion,
- $\subset$  – proper set inclusion,
- $\cup$  – union of sets,
- $\cap$  – intersection of sets,
- $\setminus$  – difference of sets,
- $+, -$  – algebraic summation and subtraction of scalars,  
vectors, or sets,
- $\text{int}(\cdot)$  – interior of a set,
- $|\cdot|$  – cardinality of a set,
- $\|\cdot\|$  – norm,
- $\text{dist}(y, A)$  – distance from element  $y$  to set  $A$ ,  $\text{dist}(y, A) = \min_{y' \in A} \|y - y'\|$

$yy'$  – scalar product; for  $y \in \mathcal{R}^k$ ,  $y' \in \mathcal{R}^k$ ,  $yy' = \sum_{i=1}^k y_i y'_i$ .