Section 1

CONTEXT OF MATHEMATICAL EDUCATION
Introduction

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Among the 14 aspects of the study identified by the International Programme Committee, the first is "context". As one can see in the discussion document, it says that "mathematics education does not take place in a vacuum, but there is always a host of different contexts within which the practice of mathematics education takes place"; moreover, these contexts may be social, political, economic, philosophical, or ethical, and they are related one way or another to the underlying cultural values.

Focusing on the aspect of "context", the first section of this study volume presents what we have achieved in this ICMI study. In general, there are three related questions to the topic of the section: (1) what are the elements within these contexts that are relevant to mathematics education? (2) What are the "givens" from which we organize and practice mathematical education? And (3) what are the "constraints" within which we carry out mathematics education?

There are seven chapters in the section. They are arranged into the following three subsections: (a) Eastern and Western Perspectives; (b) Differences and Similarities of Educational Mathematics based on Cultural History; (c) Givens and Constraints of Mathematical Education in different Cultural Traditions.

Three chapters are under the subsection of "Eastern and Western perspectives". Hirabayashi's contribution, entitled *A Traditional Aspect of Mathematics Education in Japan – Mathematics as Gei (art), its Jutsu (technique) and Do (way)*, deals with a commonly seen phenomenon not just in Japan but, in our view, also in many other parts of the world, that is, many students do not enjoy learning mathematics. The chapter examines the con-

cept and characteristics of Gei as a culture in Japanese tradition, including both Gei-Do and Gei-Jutsu, and their implication on mathematics teaching and learning in the Japanese educational settings. Hirabayashi argues that if mathematics education has some degree of the spirit of GEI, mathematics will be joyfully learned by all young pupils, or at least, will not be so much hated as now.

One can see that, interestingly, Hirabayashi's view is to a degree echoed by Ueno in the following chapter: *From Wasan to Yozan: Comparison between Mathematical Education in the Edo Period and the One after the Meiji Restoration.* In this chapter, Ueno discusses how Japanese accepted western mathematics (yozan) in the Meiji period, although in the Edo period Japanese mathematics (wasan) had bloomed. According to Ueno, in the late Edo period all over Japan people enjoyed solving problems and making new problems, but today, this tradition has almost disappeared since mathematical educators have forgotten to prepare interesting mathematical problems. Instead, students in Japan learn mathematics because it is useful for entrance examinations, while logical thinking is still missing in Japanese society and this makes mathematical education more difficult.

It has been well known since the 1980s that Japanese mathematics education has been widely acclaimed in international comparisons, especially in the West. Nevertheless, it seems to us that because of many constraints and limitations one can easily see concerning these comparisons, it is not unusual that the conclusions are often made based on limited observations, quick snapshots, and decontextualized figures and statistics, which often, though understandably, lead to misinterpretation, over-simplification, and over-generalisation. For example, we have seen that the practices about Chinese and Japanese education have been described and viewed very differently by researchers in these two countries and in the West. Therefore, more exchanges and collaborations are clearly needed between "insiders" and "outsiders" in this connection. We hope and believe that the pictures of Japanese mathematics education described in these two chapters by the two "insiders", who show a strong sense of "self-criticism" and "self-reflection" on Japanese mathematics education, will bring interested readers an enlightening perspective to think about and learn from the experiences of Japanese mathematics education, and moreover, about relevant issues concerning comparative study.

In a large sense, Keitel's contribution: *Perceptions of Mathematics and Mathematics Education in the Course of History: A Review of Western Perspectives*, is a response and dialogue with regard to the first two chapters from a Western perspective, as the subtitle suggests. Keitel looks at mathematics and mathematics education mainly as social phenomena, and thus tries to establish a more direct link to mathematics education as a social

task. For example, she argues that "rigid examinations and tests have been developed as assessment instruments for military and economic purposes first". This perspective provides us with a broad viewpoint of issues concerning examinations or testing in mathematics education. The discussion in the chapter suggests that in many aspects social structures and social needs are main determinants for mathematics education.

Fischer's chapter compares four classical ancient mathematics textbooks from different times, from different cultures, written in different languages – China's *Jiuzhang Suanshu* (about 150 B.C.) and *Suanfa Tongzong* (1592), Japan's *Jinko-ki* (1627), and a German one, *Rechnen auf den Linien und mit der Feder* (1522). The chapter aims to use historical topics as indicators for the existence of fundamentals in educational mathematics and highlight the similarities and differences in these four textbooks from a perspective of intercultural comparison. Fischer concludes that conformities between the East and the West enable us to understand each other, while differences present us with the chance to enrich and to complete each other.

The last subsection, "Givens and constraints of mathematical education in different cultural traditions", contains the last three chapters in this section. In Wong's chapter *From "Entering the Way" to "Exiting the Way": In Search of a Bridge to Span "Basic Kkills" and "Process Abilities"*, the author tries to explain and understand the phenomenon of Chinese mathematics education by examining the practice of mentoring in various Chinese traditions such as calligraphy, martial arts (Kung Fu), acupuncture, and Buddhism, and thus rejuvenating the use of practices ("entering the way") in the gradual deve-lopment of understanding and higher-order learning ("leaving the way"). According to Wong, meritable practices play a key role in enhancing students' learning of mathematics. In particular, mere repetitive practice may lead to learning by rote, but repetition with variations may lead to under-standing.

In the following chapter, Li argues that a key belief in China about teaching and learning of mathematics is that *Practice Makes Perfect.* The chapter identifies and discusses four sources of this belief: mathematics, learning, teaching and examination. The chapter also discusses two levels of meaning of manipulation or practice in practical teaching in China.

In the last chapter, Leu & Wu present an empirical study, which explored students' awareness of their teachers' mathematics pedagogical values in two Taiwanese elementary classes, with the teacher in one class being mainly influenced by Confucianism and the other by Buddhism. According to the study, Leu and Wu argue that Confucianism and Buddhism did have an influence on the teachers' mathematics pedagogical values, and students could be aware of these values as well. The chapter highlights the influence of culture on teachers' beliefs about mathematics teaching and learning and

correspondingly their teaching behaviours. The chapter also briefly discusses the implication of the findings.

We hope that readers will find the chapters contained in the section informative and enlightening.