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Volker Runde

A Taste of Topology

With 17 Figures

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Preface

If mathematics is a language, then taking a topology course at the undergraduate level is cramming vocabulary and memorizing irregular verbs: a necessary, but not always exciting exercise one has to go through before one can read great works of literature in the original language, whose beauty eventually—in retrospect—compensates for all the drudgery.

Set-theoretic topology leaves its mark on mathematics not so much through powerful theorems (even though there are some), but rather by providing a unified framework for many phenomena in a wide range of mathematical disciplines. An introductory course in topology is necessarily concept heavy; the nature of the subject demands it. If the instructor wants to flesh out the concepts with examples, one problem arises immediately in an undergraduate course: the students don't yet have a mathematical background broad enough that would enable them to understand "natural" examples, such as those from analysis or geometry. Most examples in such a course therefore tend to be of the concocted kind: constructions, sometimes rather intricate, that serve no purpose other than to show that property XY is stronger than property YX whereas the converse is false. There is the very real danger that students come out of a topology course believing that freely juggling with definitions and contrived examples is what mathematics—or at least topology—is all about.

The present book grew out of lecture notes for Math 447 (Elementary Topology) at the University of Alberta, a fourth-year undergraduate course I taught in the winter term 2004. I had originally planned to use [Simmons 63] as a text, mainly because it was the book from which I learned the material. Since there were some topics I wanted to cover, but that were not treated in [Simmons 63], I started typing my own notes and making them available on the Web, and in the end I wound up writing my own book. My audience included second-year undergraduates as well as graduate students, so their mathematical background was inevitably very varied. This fact has greatly influenced the exposition, in particular the selection of examples. I have made an effort to present examples that are, firstly, not self-serving and, secondly, accessible for students who have a background in calculus and elementary algebra, but not necessarily in real or complex analysis.

It is clear that an introductory topology text only allows for a limited degree of novelty. Most topics covered in this book can be found in any other book on the subject. I have thus tried my best to make the presentation as fresh and accessible as possible, but whether I have succeeded depends very much on my readers' tastes. Besides, in a few points, this books treats its material differently than—to my knowledge, at least—any other text on the subject.

- Baire's theorem is derived from Bourbaki's Mittag-Leffler theorem;
- Nets are extensively used, and, in particular, we give a fairly intuitive proof—using nets—of Tychonoff's theorem due to Paul R. Chernoff [CHERNOFF 92];
- The complex Stone–Weierstraß theorem is obtained via Silvio Machado's short and elegant approach [MACHADO 77].

With a given syllabus and a limited amount of classroom time, every instructor in every course has to make choices on what to cover and what to omit. These choices will invariably reflect his or her own tastes and biases, in particular, when it comes to omissions. The topics most ostensibly omitted from this book are: filters and uniform spaces. I simply find nets, with all the parallels between them and sequences, far more intuitive than filters when it comes to discussing convergence (others may disagree). Treating uniform spaces in an introductory course is a problem, in my opinion, due to the lack of elementary, yet natural, examples that aren't metric spaces in the first place.

Any book, even if there is only one author named on the cover, is to some extent an accomplishment of several people. This one is no exception, and I would like to thank Eva Maria Krause for her thorough and insightful proofreading of the entire manuscript. Of course, without my students—their feedback and enthusiasm—this book would not have been written. I hope that taking the course was as much fun for them as teaching it was for me, and that they had *A Taste of Topology* that will make their appetite for mathematics grow in the years to come.

Volker Runde
Edmonton, March 14, 2005

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