Exploring Probability in School

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Graham A. Jones
(Editor)

Exploring Probability in School

Challenges for Teaching and Learning

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CONTENTS

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GRAHAM A. JONES

PREFACE

During 1973 when I was undertaking my doctoral dissertation on young children's probabilistic thinking, I wrote to the late Efraim Fischbein to ask him some questions about my own study and to clarify some of his own work on probabilistic intuitions. In those days before the e-mail and fax technology, I was delighted to receive a very warm letter from him in less than a month. In addition to congratulating me on investigating probabilistic thinking and responding to my questions, he said in a gracious yet forthright manner that he wished that my research had involved some teaching. He went on to say that teaching probability involved special challenges because of the connection that needed to be made between theoretical and experimental notions of probability. He thought that making this connection with young children would be especially exciting and he urged me to take up the challenge in my later research.

In a very real sense, I have harbored Fischbein's challenge that has been repeated many times through his prodigious and thoughtful writing. For example, in talking about the need for an instructional program that involved both theoretical and experimental activities Fischbein and Gazit wrote, "It is that reciprocal dynamic of theoretically computed probabilities and observed relative frequencies that may best contribute to the development of efficient probabilistic intuition" (1984, p. 3).

As well as emphasizing the importance of the connection between theoretical and experimental probability, Fischbein also raised other issues about the challenges of teaching probability such as dealing with the primary intuitions that students brought to the classroom and providing appropriate representations to make probability concepts more accessible.

Accordingly this book is a response to Fischbein's enduring research and devotion to the challenges of teaching and learning probability. In initiating and editing *Exploring Probability in School: Challenges for Teaching and Learning*, I have tried to capture not only the spirit of Fischbein's original letter to me but also to gather together the writings of an international group of researchers who have made special contributions to the teaching and

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learning of probability at all levels of the school curriculum. Even though this book is a delayed response to Fischbein's challenge, the advent of probability and statistics in the mathematics curriculum at all levels makes the publication of this book on the teaching and learning of probability most timely.

I believe that the book will have special interest to a widespread audience. The book's journey through the landscape of extensive research on the learning and teaching of probability will appeal to the growing group of researchers and graduate students in fields like mathematics education, mathematics, and psychology. Curriculum developers, teacher educators and teachers will also find much to capture their interest in the learning activities and teaching implications presented at different levels of schooling. I also believe that writings on probability and people's responses to random phenomena possess a distinctive charm that hopefully will appeal to readers outside of the spheres of interest that I have foreshadowed above.

I wish to express my warmest thanks to all of the authors for their many contributions to this book. In addition to their special expertise that made each of the chapters a reality, they provided help in numerous ways to me and other authors in the team. I also want to pay special tribute to Shayne Mahon who took such wonderful pride in preparing and formatting our manuscripts through two phases of preparation. To my wife, Marion, I thank her for encouraging support over more than 40 years and for the expertise she provided in reading the chapters of this book and guiding it to a more coherent form of written communication.

I wish to add my special appreciation to Alan Bishop and members of the editorial board of the Mathematics Education Library series for their encouragement and enduring support for this project. I also thank the external reviewers who provided much valuable feedback and insight both in the proposal and review stages of this project. My sincere thanks to the editorial team at Kluwer Academic Publishers: In particular, I wish to thank Marie Sheldon, editor responsible for the Mathematics Education Library series and her assistant, Mary Panarelli, for responding to our many questions at various stages of the process and for presenting the book in such an attractive and professional manner. Finally, to Dr. John Le Blanc, Indiana University, thank you for getting me started on this creative endeavour.

1. REFERENCE

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics, 15*, 1-24.