The Construction of New Mathematical Knowledge in Classroom Interaction

Mathematics Education Library

VOLUME 38

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The Construction of New Mathematical Knowledge in Classroom Interaction

An Epistemological Perspective

Springer

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Library of Congress Cataloging-in-Publication Data

Steinbring, Heinz.
The construction of new mathematical knowledge in classroom interaction: an epistemological perspective / Heinz Steinbring.
p. cm.—(Mathematics education library; v. 38)
Includes bibliographical references and indexes.
ISBN 0-387-24251-1 (acid-free paper) -- ISBN 0-387-24253-8 (E-Book)
1. Mathematics—Study and teaching. 2. Teacher-student relationships. 3. Interaction analysis in education. I. Title. II. Series.

QA11.2.S84 2005
510'.71—dc22 2004061914

Printed in the United States of America.

9 8 7 6 5 4 3 2 1 SPIN 11347412

springeronline.com

Dedication

This book is dedicated to
Christel, Kristin and Hanna

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ACKNOWLEDGEMENTS

First of all I am especially grateful to Anna Sierpinska for her professional support in organising and accompanying the editorial work of the book. On the basis of the work of her student, Varda Levy, who polished and edited the English language as a native speaker, Anna made corrections and suggestions for improvement from the point of view of a mathematics education researcher.

I have also to mention and to thank Kristin Steinbring, my daughter, for her help in preparing a first German-English version of the book.

Without the help of Kristin, Varda Levy and Anna Sierpinska the book would not have been produced in this form. Thank you very much.

PREFACE

Mathematics is generally considered as the only science where knowledge is uniform, universal, and free from contradictions. „Mathematics is a social product – a 'net of norms', as Wittgenstein writes. In contrast to other institutions – traffic rules, legal systems or table manners –, which are often internally contradictory and are hardly ever unrestrictedly accepted, mathematics is distinguished by coherence and consensus. Although mathematics is presumably the discipline, which is the most differentiated internally, the corpus of mathematical knowledge constitutes a coherent whole. The consistency of mathematics cannot be proved, yet, so far, no contradictions were found that would question the uniformity of mathematics" (Heintz, 2000, p. 11).

The coherence of mathematical knowledge is closely related to the kind of professional communication that research mathematicians hold about mathematical knowledge. In an extensive study, Bettina Heintz (Heintz 2000) proposed that the historical development of formal mathematical proof was, in fact, a means of establishing a communicable „code of conduct" which helped mathematicians make themselves understood in relation to the truth of mathematical statements in a coordinated and unequivocal way.

If this is how one thinks about mathematical knowledge and its communication then the question for mathematics education is if this philosophical position determines some definite mental model for the teaching and learning of mathematics. Are the processes of communication in mathematical instruction comparable to those held amongst professional mathematicians? Can they or should they even conform to the experts' forms of argumentation?

In this book, I take the following approach to this problem. First of all, I assume that students in mathematics instruction are only „on their way" to becoming persons who communicate and argue mathematically. For a professional mathematician, the content related mathematical ways of argumentation are forms of communication that have become familiar through long experience. For children learning mathematics, however, mathematical argumentation represents a central content of their learning. Furthermore, the mathematical argumentation of young students is constrained by the epistemological conditions of mathematical knowledge. For students who are only beginning to learn mathematics, this knowledge is not accessible in the same abstract form as it is for professional mathematicians. It is situated and bound to concrete experiences; Bauersfeld speaks here of „subjective domains of experience" (Bauersfeld, 1983).

A second, essential aspect that has to be emphasized is that, unlike in professional mathematical communication, communication between teacher and students in the context of mathematical instruction is determined by the intention of mediat-

ing and learning mathematical knowledge. The influence of instructional goals must be particularly taken into consideration in understanding and analyzing the specificity of interactive mathematical teaching and learning processes.

In this book I present the basic concepts of the theoretical framework and I outline the chosen research methodology, which I then illustrate by a range of empirical case studies of elementary school mathematics instruction, where I analyze the conditions of the interactive construction of new mathematical knowledge. This research points at two central dimensions in the interactive knowledge construction: (1) the (general) *communicative* dimension, and (2) the *epistemological* dimension of mathematical knowledge. Interactive constructions of new mathematical knowledge as well as the necessary generalizing justifications cannot be carried out by elementary school students with the „classical" mathematical concepts of elementary algebra. This means that children can rely neither on a general algebraic notation nor on the rules of algebra in describing the yet unfamiliar knowledge or in operating with it. In the frame of elementary school mathematics instruction, new knowledge, in its interactive development, is characteristically bound to the students' situated learning and experience contexts. Children have to learn – and are able to do so by their own means – to see the general (the new knowledge) in the particular.

The central concern of this book is to investigate more closely the particularities and the variety of children's interactive constructions and justifications of mathematical knowledge in everyday mathematics teaching. The analysis of communicative factors in the culture of mathematical instruction is built on a broad theoretical basis, where the epistemological conditions of mathematical knowledge are particularly related to interactive constructions of knowledge. This is completed by detailed, extensive case studies of teaching and learning processes in which mathematical knowledge is constructed. These cases are analyzed with an epistemology-oriented methodology and the outcome of these analyses shows the complexity of the forms of constructing and justifying mathematical knowledge in instructional interactions. They provide productive possibilities, but also point to the restricting constraints of these communications. The differences between the mathematical communication of students who learn and the unequivocal communications of professional mathematicians become clear in a very evident way. However, in spite of all differences and the missing (but also not to be expected) formal, abstract argumentation, it can be observed in many teaching episodes that children participate in true mathematical communications, especially when one takes into account the particular conditions of the situatedness of the mathematical knowledge and the influences of the instructional intention within the communication.

The present book is based on a foundational theory in mathematics education about the epistemology and learning of mathematical knowledge, developed over many years of theoretical work and discussion, especially with colleagues and friends at the Institute for Didactics of Mathematics at the University of Bielefeld. Furthermore, the empirical results reported in the book are gathered in a long-term research project on mathematics teaching and learning in elementary school. This research aimed at examining the use of the substantial mathematical learning environments (from the „mathe 2000" project) in the everyday mathematics classroom. The research project was funded by the German Research Community (Deutsche Forschungsgemeinschaft, DFG; topic: „Epistemological and Socio-Interactive

Conditions of the Construction of Mathematical Knowledge Structures (in Primary Teaching)", 1.4. 1997 until 31.3.2000; reference number: STE 491/5-1 & 5-2; cf. Steinbring, 2000).

At this point, I would like to thank the participating teachers for agreeing to actively co-operate in the research project, besides having to fulfill their everyday teaching duties. I thank them for preparing the partly unfamiliar new teaching units, for carrying them out it their classes and for letting their instruction be audio- and videotaped. Without these teachers' willingness and engagement, this research project would not have been possible.