S*pringer* **M***onographs in* **M***athematics*

Springer
New York
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Functional Analysis in Mechanics

Springer

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Mathematics Subject Classification (2000): 46-01, 35-01, 74-01, 76-01, 74Kxx, 35Dxx, 35Qxx

Library of Congress Cataloging-in-Publication Data
Lebedev, L.P
Functional analysis in mechanics / L.P. Lebedev, I.I. Vorovich.
p. cm. — (Springer monographs in mathematics)
Includes bibliographical references and index.

1. Functional analysis. I. Lebedev, Leonid Petrovich, 1946– II. Title. III. Series.
QA320 .L3483 2002
515′.7—dc21 2002075732

Printed on acid-free paper.

ISBN 978-1-4419-3035-4 ISBN 978-0-387-22725-2 (eBook)
DOI 10.1007/978-0-387-22725-2

© 2003 Springer-Verlag New York, Inc.

Softcover reprint of the hardcover 1st edition 2003

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9 8 7 6 5 4 3 2 1 SPIN 10881937

Typesetting: Pages created by the authors using a Springer LaTeX 2e macro package.

www.springer-ny.com

Springer-Verlag New York Berlin Heidelberg
A member of BertelsmannSpringer Science+Business Media GmbH

Preface to the English Edition

This book started about 30 years ago as a course of lectures on functional analysis given by a youthful Prof. I.I. Vorovich to his students in the Department of Mathematics and Mechanics (division of Mechanics) at Rostov State University. That course was subsequently extended through the offering, to those same students, of another course called Applications of Functional Analysis. Later, the courses were given to pure mathematicians, and even to engineers, by both coauthors. Although experts in mechanics are quick to accept results concerning uniqueness or non-uniqueness of solutions, many of these same practitioners seem to hold a rather negative view concerning theorems of existence. Our goal was to overcome this attitude of reluctance toward existence theorems, and to show that functional analysis does contain general ideas that are useful in applications. This book was written on the basis of our lectures, and was then extended by the inclusion of some original results which, although not very new, are still not too well known.

We mentioned that our lectures were given to students of the Division of Mechanics. It seems that only in Russia are such divisions located within departments of mathematics. The students of these divisions study mathematics on the level of mathematicians, but they are also exposed to much material that is normally given at engineering departments in the West. So we expect that the book will be useful for western engineering departments as well.

This book is a revised and extended translation of the Russian edition of the book, and is published by permission of editor house Vuzovskskaya Kniga, Moscow. We would like to thank Prof. Michael Cloud of Lawrence

Technological University for assisting with the English translation, for producing the LaTeX files, and for contributing the problem hints that appear in the Appendix.

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Preface to the Russian Edition

This is an extended version of a course of lectures we have given to third and fourth year students of mathematics and mechanics at Rostov State University. Our lecture audience typically includes students of applied mechanics and engineering. These latter students wish to master methods of contemporary mathematics in order to read the scientific literature, justify the numerical and analytical methods they use, and so on; they lack enthusiasm for courses in which applications appear only after long uninterrupted stretches of theory. Finally, the audience includes mathematicians. These listeners, already knowing more functional analysis than the course has to offer, are interested only in applications. In order to please such a diverse audience, we have had to arrange the course carefully and introduce sensible applications from the beginning. The brevity of the course — and the boundless extent of functional analysis — force us to present only those topics essential to the chosen applications. We do, however, try to make the course self-contained and to cover the foundations of functional analysis.

We assume that the reader knows the elements of mathematics at the beginning graduate or advanced undergraduate level. Those subjects assumed are typical of most engineering curricula: calculus, differential equations, mathematical physics, and linear algebra. A knowledge of mechanics, although helpful, is not necessary; we wish to attract all types of readers interested in the applications and foundations of functional analysis. We hope that not only students of engineering and applied mechanics will ben-

efit, but that some mathematicians or physicists will discover tools useful for their research as well.

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